Group: ______________ ______________ ______________ Name: ______________

ma138 Worksheet 10, September 28th, 2017

An differential equation is called *autonomous* if it can be written in the form

$$\frac{dy}{dx} = g(y).$$

An *equilibrium solution* of an autonomous differential equation is a constant function which is a solution to the differential equation. In other words, it is a solution of the form $y = \hat{y}$ for some number $\hat{y}$. Because the slope of a constant function is zero, $y = \hat{y}$ is an equilibrium solution if and only if $g(\hat{y}) = 0$. An equilibrium solution is called *stable* if $g'(\hat{y})$ is negative, and *unstable* if $g'(\hat{y})$ is positive.

For each of the differential equations below, do the following steps:

a) Find the equilibrium solutions for the differential equation.

b) Draw the region of the phase plane with $x \geq 0$ and $y \geq 0$, and draw the equilibrium solutions. Do not sketch any of the vectors in the phase plane.

c) On your picture, identify all regions where the trajectory curves have positive slopes, and the regions where they have negative slope. Based on your picture, classify the equilibrium solutions as stable or unstable.

d) Use the definition at the top of the worksheet to classify the equilibrium solutions, and make sure your classifications match with part (c).

e) Draw the phase line for the differential equation.

1.

$$\frac{dy}{dx} = (y-2)(y-4)$$

2.

$$\frac{dy}{dx} = (y-1)(y-3)(y-7).$$

3.

$$\frac{dy}{dx} = (y-2)(y-4)^2.$$

You will notice in d that one of the equilibrium solutions satisfies $g'(\hat{y}) = 0$. Describe the behavior of the trajectories near this equilibrium solution. Equilibrium solutions like this are called *semi-stable*. Does $g'(\hat{y}) = 0$ imply semi-stability? Why or why not?

4.

$$\frac{dy}{dx} = g(y)$$

Where a graph of $g(y)$ is shown (Note that y is on the horizontal axis).

Figure 1: $g(y)$