	Example 1
MA 138 – Calculus 2 with Life Science Applications Rational Functions and Partial Fractions (Section 7.3)	Evaluate the following indefinite integrals $\int \frac{5}{(3x+2)^4} dx;$ $\int \frac{2x-2}{(x^2-2x+5)^3} dx.$
Alberto Corso (alberto.corso@uky.edu) Department of Mathematics	$\int (x^2 - 2x + 5)^3$
University of Kentucky Wednesday, January 25, 2017	
http://www.ms.uky.edu/*ma138 Lecture 6	http://www.ms.sky.edu/~ma138 Lecture 6
Section 7.3: Rational Functions and Partial Fractions	Proper Rational Functions
 A rational function f is the quotient of two polynomials. That is, f(x) = P(x)/Q(x) where P(x) and Q(x) are polynomials. To integrate such a function, we write f(x) as a sum of a polynomial and simpler rational functions (=partial-fraction decomposition). These simpler rational functions, which can be integrated with the methods we have learned, are of the form 	 The rational function f(x) = P(x)/Q(x) is said to be proper if the degree of the polynomial in the numerator, P(x), is strictly less than the degree of the polynomial in the denominator, Q(x), f(x) = P(x)/Q(x) proper ⇔ deg P(x) < deg Q(x). Which of the following three rational functions 3x³ - 7x² + 17x - 3
where A, B, C, a, b , and c are constants and n is a positive integer. In this form, the quadratic polynomial $ax^2 + bx + c$ can no longer be factored into a product of two linear functions with real coefficients.	$f_1(x) = \frac{3x^3 - 7x^2 + 17x - 3}{x^2 - 2x + 5}$ $f_2(x) = \frac{x}{x + 2}$ $f_3(x) = \frac{2x - 3}{x^2 + x}$ is proper? Only $f_3(x)$ is proper. The first step in the partial-fraction decomposition procedure is to use the long division algorithm to write $f(x)$ as a sum of a polynomial and a proper rational function.

Lecture 6

Lecture 6

Algebra Review

Dividing polynomials is much like the familiar process of dividing numbers. This process is the *long division algorithm for polynomials*.

Long Division Algorithm

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If A(x) and B(x) are polynomials, with $B(x) \neq 0$, then there exist unique polynomials Q(x) and R(x), where R(x) is either 0 or of degree strictly less than the degree of B(x), such that

$$A(x) = Q(x) \cdot B(x) + R(x)$$

The polynomials A(x) and B(x) are called the **dividend** and **divisor**, respectively; Q(x) is the **quotient** and R(x) is the **remainder**.

Example 2

Divide the polynomial

$$A(x) = 2x^2 - x - 4$$
 by $B(x) = x - 3$.

$$x - 3 \overline{\smash{\big)} 2x^2 - x - 4}$$

$$\underline{2x^2 - 6x}$$

$$+ 5x - 4$$

(Complete the above table and check your work!) http://www.ms.uky.edu/~ma138

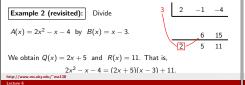
Lecture 6

Synthetic division is a quick method of dividing polynomials; it can be used when the divisor is of the form x - c, where c is a number. In synthetic division we write only the essential part of the long division table.

 In synthetic division we abbreviate the polynomial A(x) by writing only its coefficients.

Moreover, instead of B(x) = x - c, we simply write 'c.'

Writing c instead of -c allows us to add instead of subtract!



Example 3 (Online Homework # 3)

Use the Long Division Algorithm to write f(x) as a sum of a polynomial and a proper rational function

$$f(x) = \frac{x^3}{x^2 + 4x + 3}$$

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Lecture 6

Partial Fraction Decomposition (linear factors)	Example 3 (cont.d)
Case of Distinct Linear Factors	Evaluate the indefinite integral: $\int \frac{x^3}{x^2 + 4x + 3} dx$.
Q(x) is a product of <i>m</i> distinct linear factors. $Q(x)$ is thus of the form	$\int x^2 + 4x + 3 dx$
$Q(x) = a(x - \alpha_1)(x - \alpha_2) \cdots (x - \alpha_m)$	Note: from the calculations carried out in the first part of the example,
where $\alpha_1, \alpha_2,, \alpha_m$ are the <i>m</i> distinct roots of $Q(x)$. The rational function can then be written as	we know that our problem reduces to $\int (x-4) dx + \int \frac{13x+12}{(x+3)(x+1)} dx.$
$\frac{P(x)}{Q(x)} = \frac{1}{a} \left[\frac{A_1}{x - \alpha_1} + \frac{A_2}{x - \alpha_2} + \dots + \frac{A_m}{x - \alpha_m} \right]$	$\begin{bmatrix} J & J & J & (x+3)(x+1) \end{bmatrix}$
We will see in the next examples how the constants A_1,A_2,\ldots,A_m are determined.	
http://www.ms.uky.odu/"ms138 Lecture 6	http://www.ms.uky.edu/*ma138
(Heaviside) cover-up method	Example 4 (Online Homework # 8)
(Heaviside) cover-up method We illustrate this method by using the previous example:	
	Example 4 (Online Homework # 8) Find the integral: $\int_{2}^{5} \frac{2}{x^{2}-1} dx$.
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