MA162: Finite mathematics

Linear Programming: More on the Simplex Algorithm

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SCHEDULE:

Linear Programming with the Simplex Algorithm, Ex 1

Use the simplex algorithm to solve this LPP: $-3x - 4y + P = 0.$

Maximize: $P = 3x + 4y$

Constraints:

- $x + y \le 4$
- $2x + y \le 5$
- $x \ge 0, y \ge 0$

① Introduce slack variables.

$x + y + u = 4$

$2x + y + v = 5$

② Create simplex tableau

x	y	u	v	P	Const.
1	1	1	0	0	4
2	1	0	1	0	5
-3	-4	0	0	1	0

③ Use column 2 to pivot
(most negative entry in profit row appears in column 2)

④ Form ratios of Const/Pivot column:

4/1 vs. 5/1.

Pivot on Row 1 since smaller ratio.

⑤ Use row operations

$$\begin{array}{l} \\ R_2 \mapsto R_2 - R_1 \\ \\ R_3 \mapsto R_3 + 4R_1 \end{array} \quad \begin{array}{ccccc|c} x & y & u & v & P & \text{Const.} \\ 1 & 1 & 1 & 0 & 0 & 4 \\ 1 & 0 & -1 & 1 & 0 & 1 \\ \hline 1 & 0 & 4 & 0 & 1 & 16 \end{array}$$

⑥ All entries in profit row are ≥ 0, and we have complete set of basic vectors, so we have found maximum:

Conclusion

Basic Variables : y, v, P.

Non-basic : x, u.

Solution: $x = 0$, $u = 0$.

Then $y = 4$, $v = 1$

$P = 16$.

Linear Programming with the Simplex Algorithm, Ex 2

Use the simplex algorithm to solve this LPP:

Maximize: $P = 3x + 4y + z$

Constraints:

- $3x + 10y + 5z \leq 120$
- $5x + 2y + 8z \leq 6$
- $8x + 10y + 3z \leq 105$
- $x \geq 0, y \geq 0, z \geq 0$

① Introduce Slack

$$3x + 10y + 5z + u = 120$$
$$5x + 2y + 8z + v = 6$$
$$8x + 10y + 3z + w = 105$$
$$-3x - 4y - z + P = 0$$

② Simplex Tableau

3	10	5	1	0	0	0	120
5	2	8	0	1	0	0	6
8	10	3	0	0	1	0	105
-3	-4	-1	0	0	0	1	0

$120/10 = 12$

$6/2 = 3$

$105/10 = 10.5$

③ Choose column 2 to pivot (Most negative)

④ Choose row 2 to pivot (Smallest ratio)

⑤ Row Operations.

- $R_2 \mapsto R_2/2$

3	10	5	1	0	0	0	120
2.5	1	4	0	.5	0	0	3
8	10	3	0	0	1	0	105
−3	−4	−1	0	0	0	1	0

$R_1 \mapsto R_1 - 10R_2$

$R_3 \mapsto R_3 - 10R_2$

$R_4 \mapsto R_4 + 4R_2$

x	y	z	u	v	w	P	const
−22	0	−35	1	−5	0	0	90
2.5	1	4	0	.5	0	0	3
−17	0	−37	0	−5	1	0	75
7	0	15	0	2	0	1	12.

⑥ Basic Solution: y, u, w, P basic,
x, z, v non-basic.

Set $x = z = v = 0$, Then find $y = 3$, $u = 90$
$w = 75$, $P = 12$.

This is the maximal solution since there are no more negative entries in profit row.

(If negative in ~~row~~ profit row, go back to step 3 & repeat)

Linear Programming with the Simplex Algorithm, Ex 3

Boise Lumber manufactures prefabricated houses. They offer three models, standard, deluxe, and luxury.

Each house is prefabricated and partially assembled in a factory. The final assembly is done on site.

The dollar amount of building material required, the amount of factory labor required, and the amount of on-site labor required, as well as profit per unit are

	Standard	Deluxe	Luxary
Material	6000	8000	10000
Factory labor	240	220	200
On-site labor	180	210	300
Profit	3400	4000	5000

They have $\$8,200,000$ budgeted for building materials, $218,000$ hours for factory labor, and $237,000$ labor hours for on-site labor.

How many houses of each type should they build in order to maximize their profit?

Ex 3 x = Standard, y = Deluxe, z = Luxury

$$6000x + 8000y + 10000z \le 8{,}200{,}000$$
$$240x + 220y + 200z \le 218{,}000$$
$$180x + 210y + 300z \le 237{,}000$$
$$-3400x - 4000y - 5000z + P = 0$$

① Introduce slack

$$6000x + 8000y + 10000z + u = 8{,}200{,}000$$
$$240x + 220y + 200z + v = 218{,}000$$
$$180x + 210y + 300z + w = 237{,}000$$
$$-3400x - 4000y - 5000z + P = 0$$

② Tableau:

$$\left[\begin{array}{ccccccc|c}
6000 & 8000 & 10000 & 1 & 0 & 0 & 0 & 8{,}200{,}000 \\
240 & 220 & 200 & 0 & 1 & 0 & 0 & 218{,}000 \\
180 & 210 & 300 & 0 & 0 & 1 & 0 & 237{,}000 \\
\hline
-3400 & -4000 & -5000 & 0 & 0 & 0 & 1 & 0
\end{array}\right]$$

(3) Pivot on column 3.

(4) Ratios: $8200000/10000 = 820$,

$\frac{218000}{\cancel{1090}\ 200} = 1090$ Pivot Row 3

$\frac{237000}{300} = 790$

(5) Row Ops: $\text{Row}_3 \mapsto R_3/300$,

then $R_1 \mapsto R_1 - 10000 R_3$

$R_2 \mapsto R_2 - 200 R_3$

$R_4 \mapsto R_4 + 5000 R_3$

x	y	z	u	v	w	P	Const
0	1000	0	1	0	$-33.\overline{3}$	0	300,000
120	80	0	0	1	$-.6$	0	60,000
.6	.7	1	0	0	$.00\overline{3}$	0	790
-400	-500	0	0	0	$16.\overline{6}$	1	3,950,000

Not optimal since negatives remain in profit row.

Use column 2 to pivot.

Compute ratios:

$300{,}000/1000 = 300 \leftarrow$ Smallest, so Pivot on row 1.

$60{,}000/80 = 750$

$790/.7 = 1128.57$

x	y	z	u	v	w	P	Const
0	1	0	0.001	0	$-0.0\overline{3}$	0	300
120	80	0	0	1	$-0.\overline{6}$	0	6 0000
.6	.7	1	0	0	$.00\overline{3}$	0	790
−400	−500	0	0	0	$16.\overline{6}$	1	3,950,000

$R_2 \mapsto R_2 - 80R_1$

$\longrightarrow$

$R_3 \mapsto R_3 - .7R_1$

$R_4 \mapsto R_4 + 500R_1$

x	y	z	u	v	w	P	Const.
0	1	0	0.001	0	$-0.0\overline{3}$	0	300
120	0	0	−.08	1	2	0	36,000
0.6	0	1	−0.0007	0	$0.02\overline{6}$	0	580
$\boxed{-400}$	0	0	1/2	0	0	1	4,100,000.

Still negatives, Oh No!

Still not optimal, Pivot on column 1

Form ratios: Row 1 : Don't take ratio, denom is zero

Row 2 : $36000/120 = 300 \leftarrow$ Pivot on row 2.

Row 3 : $580/.6 = 966.\overline{6}$

Do $R_2 \mapsto R_2/120$

x	y	z	u	v	w	P	Const.
0	1	0	0.001	0	$-0.0\overline{3}$	0	300
1	0	0	$-0.000\overline{6}$	$0.008\overline{3}$	$0.01\overline{6}$	0	300
.6	0	1	-0.0007	0	$0.02\overline{6}$	0	580
-400	0	0	.5	0	0	1	4,100,000

$R_3 \mapsto R_3 - .6R_2$

$R_4 \mapsto R_4 - 400R_2$

0	1	0	0.001	0	$-0.0\overline{3}$	0	300
1	0	0	$-0.000\overline{6}$	$0.008\overline{3}$	$0.01\overline{6}$	0	300
0	0	1	-0.0003	-0.005	$0.01\overline{6}$	0	400
0	0	0	$0.2\overline{3}$	$3.\overline{3}$	$6.\overline{6}$	1	4,220,000.

FINALLY!

Final recommendation:

Basic: x, y, z, P

Non-basic: u, v, w

Basic Solution: $u = v = w = 0$

$u = 0 \Rightarrow$ No unused material

$v = 0 \Rightarrow$ No unused fact. labor

$w = 0 \Rightarrow$ No unused On-site labor

Solve for the values of basic:

$y = 300$

$x = 300$

$z = 400$

$P = 4{,}220{,}000$

Make 300 standard, 300 deluxe, 400 luxury models. This results in \$4,220,000 in profit and ~~no~~ no unused resources.