MA162: Finite mathematics

Linear Programming: More on the Simplex Algorithm

Paul Koester

University of Kentucky

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Solutions

SCHEDULE:

Linear Programming with the Simplex Algorithm

A farmer has 100 acres of land suitable for growing corn, wheat, and soy. Each acre of corn costs \$600 to grow and requires 40 hours of labor. Each acre of wheat costs \$550 to grow and requires 32 hours of labor. Each acre of soy costs \$500 to grow and requires 40 hours of labor. The farmer has \$50,000 to spend on growing crops and 2000 labor hours. Profits are \$350 per acre of corn, \$325 per acre of wheat, and \$375 per acre of soy.

Help the farmer determine how much of each crop should be grown in order to maximize her profit.

Are there any left over resources? If so, how much of each?

① Introduce Variables.

C = Acres corn, W = Acres wheat, S = Acres Soy

Objective: Maximize $P = 350C + 325W + 375S$

Constraints: $C + W + S \leq 100$

$600C + 550W + 500S \leq 50000$

$40C + 32W + 40S \leq 2000.$

② Introduce Slack: u_1, u_2, u_3. (Usually use letters like u, v, w, but already using w for wheat...)

$$\begin{cases} C + W + S + u_1 = 100 \\ 600C + 550W + 500S + u_2 = 50000 \\ 40C + 32W + 40S + u_3 = 2000 \\ -350C - 325W - 375S + P = 0. \end{cases}$$

③ Create Tableau

C	W	S	u_1	u_2	u_3	P		Ratios.
1	1	1	1	0	0	0	100	$100/1 = 100$
600	550	500	0	0	0	0	50000	$50000/500 = 100$
40	32	40	0	0	1	0	2000	$2000/40 = 50$
-350	-325	-375	0	0	0	1	0	

④ Current basic solution:
Set $C = W = S = 0$, Then

$u_1 = 100$
$u_2 = 50000$
$u_3 = 2000$
$P = 0$.

Not optimal since negative values remain in profit row.

⑤ Choose new pivot:
Column 3, since largest negative value in profit row is in column 3.

Row 3 since smallest ratio

(6) Row Operations.

$R_3 \mapsto R_3/40$

1	1	1	1	0	0	0	100
600	550	500	0	1	0	0	50000
1	.8	(1)	0	0	1/40	0	50
-350	-325	-375	0	0	0	1	0

Then

	C	W	S	u_1	u_2	u_3	P	
$R_1 \mapsto R_1 - R_3$	0	0.2	0	1	0	-0.025	0	50
$R_2 \mapsto R_2 - 500R_3$	100	150	0	0	1	-12.5	0	25000
$R_3 \mapsto R_3 + 375R_3$	1	.8	1	0	0	.025	0	50
	25	-25	0	0	0	9.375	1	18750

(7) Current basic solution:

$C = W = u_3 = 0$.

$u_1 = 50$, $u_2 = 25000$, $S = 50$, $P = 18750$.

Not optimal since negative in profit row.

⑧ Choose new pivot

0	.2	0	1	0	−0.025	0	50
100	150	0	0	1	−12.5	0	25000
1	.8	1	0	0	−0.025	0	50
25	−25	0	0	0	9.375	1	18750

Ratios

$50/.2 = 250$

$25000/150 = 166.7$

$50/.8 = 62.5$

- Column 2 since most negative entry in profit row is in column 2.
- Row 3 since smallest ratio in Row 3

⑨ Row Operations:

$R_3 \mapsto R_3/.8$

0	.2	0	1	0	−0.025	0	50
100	150	0	0	1	−12.5	0	25000
1.25	1	1.25	0	0	−0.03125	0	62.5
25	−25	0	0	0	9.375	1	18750

Next,

	C	W	S	u_1	u_2	u_3	P	
	-.25	0	-.25	1	0	-.03125	0	37.5
$R_1 \mapsto R_1 - .2R_3$	-87.5	0	-187.5	0	1	-17.1875	0	15625
$R_2 \mapsto R_2 - 150R_3$	1.25	1	1.25	0	0	-0.03125	0	62.5
$R_4 \mapsto R_4 + 25R_3$	56.25	0	31.25	0	0	10.15625	1	20312.5

(10) Current basic solution

$C = S = u_3 = 0$

Then $u_1 = 37.5$

$u_2 = 15625$

$W = 62.5$

$P = 20312.5$

Best possible, since all entries in profit row are non-negative.

Plain English interpretation: Use 62.5 acres for wheat, Grow no corn + no soy.

This leaves 37.5 acres unused and \$15,625 left over budget. Total profit is \$20,312.5