MA162: Finite mathematics

Financial Mathematics

Paul Koester

University of Kentucky

February 3, 2014

Schedule:

Loans

- An amount \$P is borrowed. (P stands for principal, or present value)
- The loan is to be repaid by making *regular* payments of size \$R and the end of each period for the next n periods.
- Interest rate is i per period.
- Then
$$P = R \cdot \frac{1-(1+i)^{-n}}{i}$$
- In Excel, P can be computed by $=$*PV(i,n,R)*.
- In WeBWorK, P can be computed by *R * PV(i,n)*.

Ex. 1: Car Loan

- Murray just purchased a car. The price of the car was $\$15,000$.
- He makes a $\$4000$ down payment takes out a car loan to cover the rest.
- He has to make payments at the end of each month for the next 4 years.
- The interest on the loan is 6% APR compounded monthly.
- Determine the size of Murray's monthly payment.

Ex. 1: Car Loan (Continued)

- What is the total amount of interest that Murray pays?

- How much of Murray's first payment is due to interest?

Ex. 1: Car Loan (Continued)

- It is now 2.5 years from the time Murray took out his car loan and Murray just made the 30^{th} payment on his car.
- How much would he need to pay now in order to pay off the rest of his loan[1]?
- What is the total amount of interest that Murray pays assuming he pays off the balance in full immediately after the 30^{th} payment?

[1] assuming no "early pay-off fees"

Ex. 2: Home-a-loan

- Norah has a 15 year home mortgage.
- She needs to pay \$2300 at the end of each month for the next 15 years.
- The interest on the loan is 3.625% APR compounded monthly.
- She is having trouble affording the \$2300 per month.
- To lower her monthly payment, she is going to refinance to a 30 year loan which has 4.5% APR compounded monthly.

Ex. 2: Home-a-loan

- Determine the size of her new monthly payment.

Ex. 2: Home-a-loan

- Determine the total interest charges on the original loan.

- Determine the total interest charges on the new loan.

Ex. 3: Chance to buy a ranch

- Blanch can't pass up the chance to buy a ranch.
- She will borrow $400,000.
- She will pay back this loan by making quarterly payments at the end of each quarter for 30 years.
- Interest on the loan is 6.2% APR compounded quarterly.

- Determine the size of Blanch's quarterly payments.

- Determine the interest charges on the loan.

Ex. 3: Chance to buy a ranch

- Blanch suspects she can drastically cut her interest expenses if she is able to make quarterly payments that are larger than required.
- Supposing that Blanch pays twice her scheduled payment each month, determine how many payments Blanch needs to make before she pays off the loan.
- Determine Blanch's interest charges on the loan if she makes double payments.

Annuities

- A sequence of regular cash flows of $R occurs at the end of each period for the next n periods. (R stands for "regular cash flow")
- Interest rate is i per period.
- Then the present value, P, of this annuity is

$$P = R \cdot \frac{1 - (1+i)^{-n}}{i}$$

- In Excel, P can be computed by *=PV(i,n,R)*.
- In WeBWorK, P can be computed by *R * PV(i,n)*.
- P answers the question "What is the value of this entire stream of cash flows evaluated at the beginning"

Annuities

- Then the accumulated value, or future value, F, of this annuity is
$$F = R \cdot \frac{(1+i)^n - 1}{i}$$
- In Excel, P can be computed by *=FV(i,n,R)*.
- In WeBWorK, P can be computed by *R * FV(i,n)*.
- F answers the questions like "If you save $R at the end of each year for the next n years and interest is i per year, then what is the value of your savings at the end?"

Annuities versus Loans

- Annuities and loans both involve level sized cash flows that are paid at regular time intervals
- Mathematically, they are treated the same
- Financially, the regular cash flows in a loan are being paid out, while the regular cash flows in an annuity are being received

Ex 4: FV of Annuity

- Determine the accumulated value of a 8 year annuity with level cash flows of \$1200 at the end of each quarter, provided the cash flows earn 6% annual interest compounded quarterly.

- Determine the present value of the above annuity.