MA162: Finite mathematics

Financial Mathematics

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SCHEDULE:

Annuities

- A sequence of regular cash flows of \$R occurs at the end of each period for the next n periods. (R stands for "regular cash flow"))
- Interest rate is i per period.
- Then the present value, P, of this annuity is

$$P = R \cdot \frac{1 - (1+i)^{-n}}{i}$$

- In Excel, P can be computed by *=PV(i,n,R)*.
- In WeBWorK, P can be computed by *R * PV(i,n)*.
- P answers the question "What is the value of this entire stream of cash flows evaluated at the beginning"

Annuities

- Then the accumulated value, or future value, F, of this annuity is

$$F = R \cdot \frac{(1+i)^n - 1}{i}$$

- In Excel, P can be computed by *=FV(i,n,R)*.
- In WeBWorK, P can be computed by *R * FV(i,n)*.
- F answers the questions like "If you save $R at the end of each year for the next n years and interest is i per year, then what is the value of your savings at the end?"

Ex. 1: FV of Annuity

- Determine the accumulated value of a 8 year annuity with level cash flows of \$1200 at the end of each quarter, provided the cash flows earn 6% annual interest compounded quarterly.

$R = 1200, \quad n = 8 \cdot 4 = 32, \quad i = 0.06/4 = 0.015$

$$F = 1200 \frac{(1.015)^{32} - 1}{0.015}$$

$$= \$48,825.95$$

- Determine the present value of the above annuity.

$$P = 1200 \frac{1 - (1.015)^{-32}}{.015} = \$30,320.56.$$

Alternatively,

$$\frac{48,825.95}{(1.015)^{32}} = \$30,320.56$$

Ex 2: PV of Annuity

$R = \$45{,}000$

$i = .05$

$n = 30$

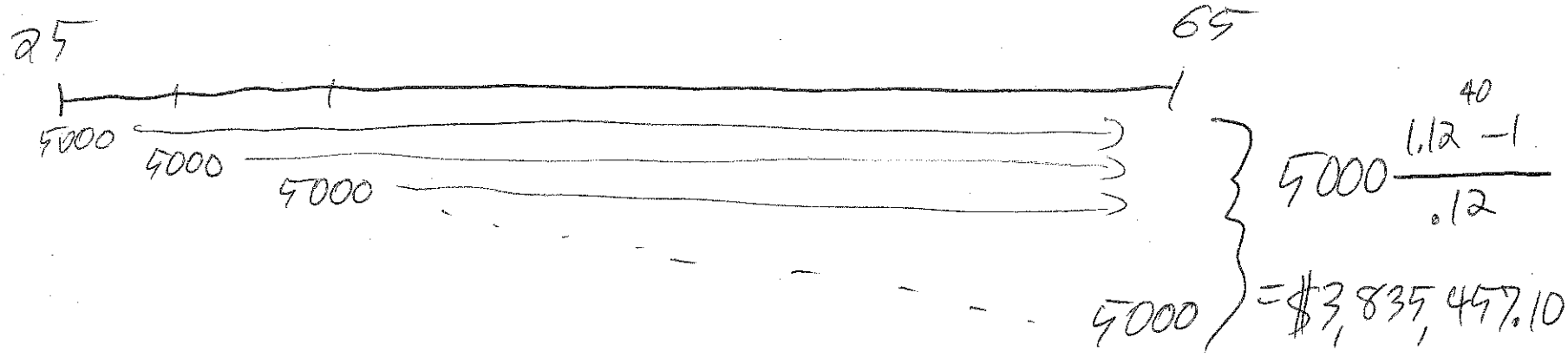
- Orion was involved in a terrible accident at work.
- Worker's Comp insurance is to pay Orion \$45,000 at the end of each year for the next 30 years.
- Orion gets a phone call from one of those companies from day-time TV ads that say they buy "structured settlements and annuities." Basically, the company will give Orion a large lump sum of cash today in exchange for his annuity.
- The company offers to give him \$550,000 today in exchange for his structured settlement.
- Orion wants to make sure they are not trying to rip him off.
- Assuming interest is 5% annually compounded, what is the fair value of his structured settlement, and should he take the \$550,000 now?

$$P = 49{,}000\,\frac{1-(1.05)^{-30}}{0.05} = \$691{,}760.30$$

Ex 3: IRA

- Gilligan is 25 years old. He just graduated with his MBA and got a good job.
- He is going to start saving for retirement. He will do this by depositing \$5000 at the end of each year into an IRA (Individual Retirement Account).
- Assuming the account earns 12% per year compounded annually, how much will Gilligan have in this account when he retires at age 65?

$R = 5000$, $i = 0.12$, $n = 40$ $(65-25)$.



Ex 3: IRA

- Now suppose Gilligan decides to wait until he's 30 to start saving for retirement.
- How much will he have in his retirement account when he turns 65?

Just change $n = 35$

~~5000~~ $\cdot (1.12)$

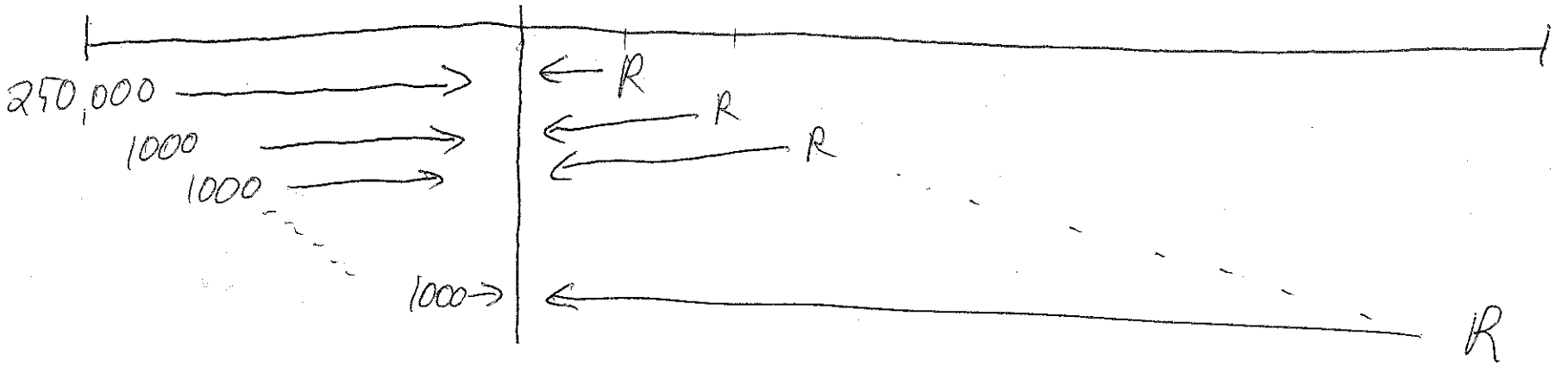
$$5000 \; \frac{1.12^{35} - 1}{.12} = \$2,158,317.48$$

Notice 5 year difference results in change of $\approx \$1,700,000$!

Ex 4: Myra's early retirement

- Myra is 40 years old. She wants to retire early, when she turns 55.
- She currently has $250,000 in an investment account.
- She figures she can deposit $1000 at the end of each month from now until the time she retires.
- Her investment account will earn 12% interest per year compounded monthly.
- When she retires, she will move her investment account to a safer but lower yielding account which will earn 3% per year compounded monthly.
- She figures her retirement funds must last until she turns 85.
- How much will she be able to withdraw from the account at the end of each month during her retirement?

40 55 85



$$250{,}000\left(1+\frac{.12}{12}\right)^{12\cdot 15}+1000\,\frac{\left(1+\frac{.12}{12}\right)^{12\cdot 15}-1}{\left(\frac{.12}{12}\right)}$$

$$= R\cdot\frac{1-\left(1+\frac{.03}{12}\right)^{-12\cdot 30}}{\left(\frac{.03}{12}\right)}$$ rhs simplifies.

LHS simplifies to $\$1{,}998{,}530.69 = R\cdot 237.1893815$.

So $\boxed{R = \$8{,}427.89}$