MA162: Finite mathematics

Introduction to Probability

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SCHEDULE: Solutions

7.1: Probability Theory Basics

- There is much overlap between Counting (Chapter 6) and Probability Theory (Chapter 7), but some terminology is different
- An experiment is an activity with observable results
- A sample point is an outcome of an experiment
- A sample space is the collection of all sample points of an experiment
- An event is a collection of some (possibly all, possibly none) sample points of an experiment
- A simple event is an event containing exactly one sample point

7.1: Counting to Probability Dictionary

Counting		**Probability**
element of a set	$\leftrightarrow$	sample point
universal set	$\leftrightarrow$	sample space
subset of a set	$\leftrightarrow$	event

Example of a sample space

- The experiment: You flip a quarter and a dime, and observe whether heads or tails on each coin.
- What is the sample space?

{ (H, H), (H, T), (T, H), (T, T) }

Quarter Dime.

- What are the simple events?

(H, H),

(H, T)

(T, H)

(T, T).

The Empirical Probability

Given an event E in a sample space S, the empirical probability, or relative frequency, of E is given by

$$P(E) = \frac{n(E)}{n(S)}$$

Think of this as "probability is given by number of favorable outcomes divided by total number of outcomes."

- The experiment: You flip a quarter and a dime, and observe whether heads or tails on each coin.
- What is the probability you observe at least one heads?

Assume fair coins, so each simple event equally likely

(H, H) (T, H)

(H, T) (T, T)

$\frac{3}{4}$.

- What is the probability the dime shows a heads?

Q D

(H, H) (T, H)

(H, T) (T, T)

$\frac{2}{4} = \frac{1}{2}$.

Probability Spaces

A pair of fair six sided dice are rolled.
Determine an appropriate sample space.

Dice

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

a.) Determine the event: One die has an odd number and one die has either the number 1 or 2.

Include all with 1 (satisfies both)
Then include all with 2 & an odd.

b.) Determine the probability of the above event.

$$\frac{15}{36}$$

c.) A pair of fair six sided dice is rolled. Determine the probability that the sum of the values of the dice is exactly 4.

How to get a 4? $\left.\begin{array}{l} 1+3=4 \\ 2+2=4 \\ 3+1=4 \end{array}\right\}$ 3 ways

$\frac{3}{36}$

d.) A pair of fair six sided dice is rolled. Determine the probability that the sum of the value of the dice is at least 4.

At least 4 $\Rightarrow$ (Exactly 4) $\cup$ (Exactly 5) $\cup$ (Exactly 6) etc. .....

Lots!!

Easier approach: There are only $\underline{3}$ that don't count:

$1+1=2$, $1+2=3$, $2+1=3$

All $-$ those that don't apply.

So # that are at least 4 $= 36 - 3 = 33$

So Prob (At least a 4) $= \frac{33}{36}$

Cards

52 cards
13 diamonds
4 "5"s
4 "Kings"

- One card is drawn at random from a standard deck of 52 cards.
- Let $A = \{$ card is a "Diamond" $\}$, $B = \{$ card is a "5" $\}$, $C = \{$card is a "King"$\}$.
- $P(A) =$ $13/52 = 1/4$.
- $P(B) =$ $4/52 = 1/13$
- $P(C) =$ $4/52 = 1/13$.
- $P(B \cup C) =$ $8/52$. ← 4 "5" + 4 "King"
- $P(A \cap B) =$ $1/52$. ← Only one 5 of Diamond
- $P(A \cup B) =$ $P(A) + P(B) - P(A \cap B)$ $= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52}$

  card is Diamond; card is 5; 5 of Diamonds overcounting.
- $P(B \cap C) =$ $0/52 = 0$

can't be "5" & "King"

Mutually Exclusive Events

- Two events are Mutually Exclusive if they cannot occur simultaneously
- In other words, $E_1 \cap E_2 = \emptyset$

Empirical Distribution: Example

200 students were surveyed regarding the number of credit hours they are taking in the current semester. Here are their responses:

Credit hours	12 or fewer	13 hours	14 hours	15 hours	16 hours	17 hours	18 or more	
Number students	46	51	29	55	12	6	1	= 200

A student is selected at random from this group. What is the probability that they are taking at least 15 credit hours?

15, 16, 17, 18 or More.

$$\frac{55+12+6+1}{200} = \frac{74}{200}.$$

7.2: Another Empirical Distribution

Age	85	86	87	88	89	90	91
Number deaths	3590	3711	3796	3838	3828	3757	3624

The above is an excerpt from the Social Security Administrations Mortality Table. The table follows a cohort of 100,000 people and records the age of death of these individuals.

- Determine the probability a randomly chosen individual dies at age 90. $3757/100,000$
- Determine the probability a randomly chosen individual dies between age 87 and 90.

$$\frac{3796 + 3838 + 3828 + 3757}{100,000}$$