MA162: Finite mathematics

Financial Mathematics

University of Kentucky

January 29, 2014

Schedule:

Last Time

- **Compound Interest, Compounded m times per year**
  If \$P is invested for t years at annual interest rate r compounded m times per year, then A (the accumulated value) is
  $$A = P\left(1 + \frac{r}{m}\right)^{mt}$$
- Equivalently, letting $i = r/m$ (interest per period) and $n = mt$ (total number of periods) then
  $$A = P\,(1 + i)^n$$
- **Continuously Compounded interest**
  If \$P is invested for t years at continuously compounded rate of δ per year, then
  $$A = P\,e^{\delta t}$$

Last Time

- Consider annual compound interest r compounded m times per year. The *annual effective rate of interest*, r_eff is obtained by solving this equation for r_{eff} :

$$\left(1 + \frac{r}{m}\right)^m = (1 + r_{eff})$$

- Consider continuously compounded interest rate of δ. The *annual effective rate of interest*, r_eff is obtained by solving this equation for r_{eff} :

$$e^\delta = (1 + r_{eff})$$

Ex. 1: Continuous Compound Interest

Find the present value of \$2700 invested for 12 years at continuously compounded rate of 2%.

Ex. 2: Effective Rate

Determine the continuously compounded rate of interest if the effective rate of interest is 6%

Ex. 3: Dealing with Varying Rate

- John borrowed \$3500 on January 29, 2010. He is to repay the loan on January 29, 2014.
- His loan is an adjustable rate loan, meaning that the interest rate *can* vary at certain specific times.
- The original interest rate was 3% APR compounded quarterly.
- On July 29, 2013, the interest rate was changed to 5%.
- Assuming no more changes in the interest rate, how much must he pay back?

Ex. 4: Dealing with multiple cash flows

- Kendall opened a savings account 6 years ago, by invested \$5000 in an account which earns 4% APR compounded semi-annually.
- 5 years ago, she deposited an extra \$2000.
- 2 years ago, she withdrew some amount, call it X.
- Her current balance is \$4361.
- Determine the size of X.

Ex. 5: Dealing with lots of cash flows

- Loraine won the Ohio Lottery. She can choose between accepting \$9,000,000 today or she can accept \$600,000 at the end of each year for the next 30 years.
- Assume discount rate of 3% per year, compounded annually.
- Which option is "better"?

Geometric Sum

Given a real number $a \neq 1$ and a positive integer n,

$$a + a^2 + a^3 + \ldots + a^n = \frac{a - a^{n+1}}{1 - a}$$

Consequently,

$$\frac{1}{1+i} + \left(\frac{1}{1+i}\right)^2 + \left(\frac{1}{1+i}\right)^3 + \ldots + \left(\frac{1}{1+i}\right)^n = \frac{\frac{1}{1+i} - \left(\frac{1}{1+i}\right)^{n+1}}{1 - \frac{1}{1+i}}$$

and with a bit of algebra, this simplifies down to

$$\frac{1 - (1+i)^{-n}}{i}$$

Loans

- An amount \$P is borrowed. (P stands for principal, or present value)
- The loan is to be repaid by making *regular* payments of size \$R and the end of each period for the next n periods.
- Interest rate is i per period.
- Then
$$P = R \cdot \frac{1-(1+i)^{-n}}{i}$$
- In Excel, P can be computed by *=PV(i,n,R)*.
- In WeBWorK, P can be computed by *R * PV(i,n)*.

Ex. 6: Car Loan

- Murray just purchased a car. The price of the car was $\$15,000$.
- He makes a $\$4000$ down payment takes out a car loan to cover the rest.
- He has to make payments at the end of each month for the next 4 years.
- The interest on the loan is 6% compounded monthly.
- Determine the size of Murray's monthly payment.

Ex. 6: Car Loan

- What is the total amount of interest that Murray pays?

- How much of Murray's first payment is due to interest?