Exam 3

MA 162: Finite Mathematics
University of Kentucky

April 14, 2014

Directions:

- Do not remove this page—you will turn in the entire exam.
- Complete this exam using only a pen or pencil and a simple calculator (not a cellphone).
- The point value for each question is shown in the exam booklet.
- On free response questions you must show all work in order to receive credit. Unjustified answers will receive no credit!
- Please denote row operations using the notation $R_2 \mapsto R_2 + 3R_1$, etc. If we cannot follow your work, we will stop reading your work and assign 0 points to your question.
- If asked to explain, you must write clearly and in complete sentences.
- Use the back side of the exam for scrap work. You are not allowed to use your own scratch paper.

Printed Name: Solutions. **Section:** ________

Do Not Write Anything Here

Question	Points Possible	Score
Q 1	2 points	
Q 2	6 points	
Q 3	6 points	
Q 4	6 points	
Q 5	6 points	
Q 6	6 points	
Q 7	6 points	
Q 8	6 points	
Q 9	8 points	
Q 10	8 points	
Total	60 points	

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1. (3 points) The matrix on the right is obtained by performing a row operation on the matrix on the left. Determine the row operation. Write the row operation in the notation $R_3 \mapsto R_3 + 2R_4$.

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 10 & -5 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Rows 2 & 3 didn't change, so operate on Row 1.

The "3" in Row 1 column 2 is eliminated, so put Row 1 against Row 2.

$$R_1 \mapsto R_1 - 3R_2$$

2.

$$\left[\begin{array}{ccc|c} 1 & -5 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

(a.) (2 points) Does the linear system corresponding to the above matrix have any solutions? Yes No (Circle One)

(b.) (4 points) If the system does not have any solutions, explain why in complete sentences. If the system has at least one solution, apply row operations to put the matrix in row reduced form. You must show all your steps. Then interpret your answer. If the system has more than one solution, write the solution in parametrized form.

$$R_1 \mapsto R_1 + 5R_2 \quad \begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 23 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x = 23$

$y = 4$

$z = 3$

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3.

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 4 \\ 0 & 2 & 0 & 8 & 6 \\ 0 & 0 & 1 & 4 & 5 \end{array}\right] \xrightarrow{R_2 \mapsto R_2/2} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 4 & 3 \\ 0 & 0 & 1 & 4 & 5 \end{array}\right]$$

(a.) (2 points) Does the linear system corresponding to the above matrix have any solutions? Yes No (Circle One)

(b.) (4 points) If the system does not have any solutions, explain why in complete sentences. If the system has at least one solution, apply row operations to put the matrix in row reduced form. You must show all your steps. Then interpret your answer. If the system has more than one solution, write the solution in parametrized form.

$$\xrightarrow{R_1 \mapsto R_1 - 3R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -12 & -11 \\ 0 & 1 & 0 & 4 & 3 \\ 0 & 0 & 1 & 4 & 5 \end{array}\right] \qquad \begin{aligned} x - 12w &= -11 \\ y + 4w &= 3 \\ z + 4w &= 5 \end{aligned}$$

$$\begin{cases} x = 12t - 11 \\ y = -4t + 3 \\ z = -4t + 5 \end{cases}$$

4.

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 2 & -4 & 8 \\ 4 & -4 & 16 \end{array}\right]$$

(a.) (2 points) Does the linear system corresponding to the above matrix have any solutions? Yes No (Circle One)

(b.) (4 points) If the system does not have any solutions, explain why in complete sentences. If the system has at least one solution, apply row operations to put the matrix in row reduced form. You must show all your steps. Then interpret your answer. If the system has more than one solution, write the solution in parametrized form.

$$\xrightarrow[R_3/2]{R_2/2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 1 & -2 & 4 \\ 1 & -1 & 4 \end{array}\right] \xrightarrow[R_3 \mapsto R_3 - R_1]{R_2 \mapsto R_2 - R_1} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & -2 & 1 \\ 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow[R_3/-1]{R_2/-2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1/2 \\ 0 & 1 & -1 \end{array}\right] \xrightarrow{R_3 \mapsto R_3 - R_2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & -1/2 \end{array}\right]$$

Bottom row $\Rightarrow$ $0x + 0y = -1/2$

i.e, $0 = -1/2$.

NO SOLUTION.

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5. (6 points) Mr. Jones just inherited \$150,000. He will invest some of his inheritance in the stock market, some in the bond market, and he will denote the rest to charity. He will earn 12% on stocks and 5% on bonds. He earns nothing on the charitable donation. He wants the sum invested in stocks and bonds to be twice the charitable donation. He wants to earn a total of \$12,000 on the investments.
Write down a system of equations to help Mr. Jones determine how much of his inheritance should be invested in stocks, how much in bonds, and how much should be given to charity.
NOTE: IN THIS QUESTION, YOU ARE ONLY ASKED TO WRITE DOWN THE SYSTEM OF EQUATIONS. YOU ARE NOT REQUIRED TO SIMPLIFY THE SYSTEM. YOU ARE NOT REQUIRED TO SOLVE THE SYSTEM. YOU ARE NOT REQUIRED TO WRITE DOWN THE ASSOCIATED MATRIX.

$S+B=2C$

S= Amt in stocks, B= Amt in Bonds, C=Amt to charity

$$\begin{cases} S+B+C = 150{,}000 \\ .12S + .05B = 12{,}000 \\ S+B-2C = 0 \end{cases}$$

6. (6 points) Compute the matrix product $\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 4 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$ Show all work. No work = no credit!

$(3\times 2)(2\times 4) \Rightarrow$ Product has 3 rows 4 columns

Product is defined

$$\begin{bmatrix} 1\cdot1+0\cdot1 & 1\cdot1+0\cdot2 & 1\cdot1+0\cdot3 & 1\cdot1+0\cdot4 \\ 0\cdot1+2\cdot1 & 0\cdot1+2\cdot2 & 0\cdot1+2\cdot3 & 0\cdot1+2\cdot4 \\ 4\cdot1+(-3)\cdot1 & 4\cdot1+(-3)\cdot2 & 4\cdot1+(-3)\cdot3 & 4\cdot1+(-3)\cdot4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 \\ 1 & -2 & -5 & -8 \end{bmatrix}$$

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7. (6 points) Farmer Jon has 100 acres that can be used to grow corn, alfalfa, and barley. Each acre of corn costs \$625 to grow. Each acre of alfalfa costs \$525 to grow. Each acre of barley costs \$500 to grow. Each acre of corn requires 24 hours of labor. Each acre of alfalfa requires 12 hours of labor. Each acre of barley requires 9 hours of labor. Farmer Jon has a total budget of \$57,000 and has a total of 1740 hours of labor.
Farmer Jon set up a system of equations and simplified it to get

$$\begin{cases} C &= 45 + 0.25 \cdot t \\ A &= 55 - 1.25 \cdot t \\ B &= t \end{cases}$$

$C = 50 = 45 + .25t \Rightarrow 5 = \frac{1}{4}t \Rightarrow t = 20.$

where C denotes number of acres of corn, A denotes number of acres of alfalfa, and B denotes number of acres of barley. Suppose Farmer Jon wants to grow exactly 50 acres of corn. Use his above solution to determine the number of acres of alfalfa and number of acres of barley.

$C = 45 + .25 \cdot 20 = 50$

$A = 55 - 1.25 \cdot 20 = 30$

$B = 20.$

8. (6 points)

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$$

Compute $A + 2(B - C)$. Show all work. No work = no credit!

$B - C = \begin{bmatrix} 1-4 & 6-5 \\ 3-2 & 2-1 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 1 & 1 \end{bmatrix}$

So $2(B-C) = \begin{bmatrix} 2(-3) & 2 \cdot 1 \\ 2 \cdot 1 & 2 \cdot 1 \end{bmatrix} = \begin{bmatrix} -6 & 2 \\ 2 & 2 \end{bmatrix}$

$A + 2(B-C) = \begin{bmatrix} 3 & 1 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} -6 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 3-6 & 1+2 \\ 0+2 & 5+2 \end{bmatrix}$

$= \begin{bmatrix} -3 & 3 \\ 2 & 7 \end{bmatrix}$

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9. (a.) (4 points) Determine the matrix inverse of $A = \begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ by performing elimination on the augmented matrix

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2/2} \left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

. Show all work. No work = no credit.

$$\xrightarrow[R_2 \mapsto R_2 - 2R_3]{R_1 \mapsto R_1 - 3R_3} \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & -3 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \mapsto R_1 - 4R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 5 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 5 \\ 0 & \frac{1}{2} & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b.) (4 points) Solve the matrix equation

$$\begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} \qquad AX = B \Rightarrow X = A^{-1}B$$

Show all work. No work = no credit.

$$\Rightarrow X = \begin{bmatrix} 1 & -2 & 5 \\ 0 & \frac{1}{2} & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 3 + (-2)(-2) + 5(0) \\ 0 \cdot 3 + \frac{1}{2}(-2) + (-2)(0) \\ 0 \cdot 3 + 0 \cdot (-2) + 1 \cdot 0 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$$

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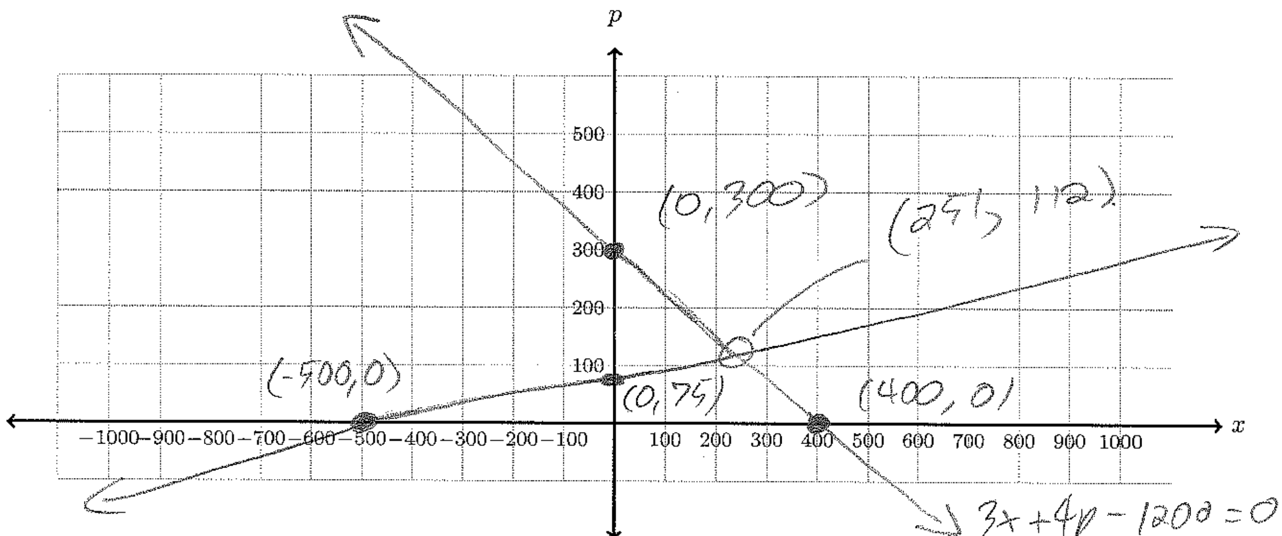
10. The demand equation for the Drake Bluetooth speaker is $3x + 4p - 1200 = 0$ where x is the quantity demanded per week and p is the wholesale unit price in dollars. The supply equation is $3x - 20p + 1500 = 0$ where x is the quantity the supplier will make available in the market each week when the wholesale unit price is p dollars each.
(a.) (3 points) Graph both the demand equation and the supply equation on the same axes below.



(b.) (3 points) Use algebra to find the equilibrium quantity and equilibrium price for the Bluetooth speakers.

$$\begin{aligned} 3x+4p&=1200 \\ -3x+20p&=1500 \end{aligned} \Rightarrow \begin{array}{cc} x & p \\ \end{array} \left[\begin{array}{cc|c} 3 & 4 & 1200 \\ -3 & 20 & 1500 \end{array}\right]$$

$$R_2 \mapsto R_2 + R_1 \left[\begin{array}{cc|c} 3 & 4 & 1200 \\ 0 & 24 & 2700 \end{array}\right] \xrightarrow[R_2/24]{R_1/3} \left[\begin{array}{cc|c} 1 & 4/3 & 400 \\ 0 & 1 & 112 \end{array}\right]$$

$$\xrightarrow{R_1 \mapsto R_1 - \frac{4}{3}R_2} \left[\begin{array}{cc|c} 1 & 0 & 251 \\ 0 & 1 & 112 \end{array}\right]$$

$x = 251$
$p = 112$

(c.) (2 points) In complete sentences, explain how your graphs in part (a) relate to the answer you attained for part (b).

Algebraically, equilibrium corresponds to simultaneous solution to system, but this geometrically corresponds to intersection point of the lines

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