MA162: Finite mathematics

Introduction to Linear Algebra

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SCHEDULE:

Linear Equations: Familiar Things

In algebra, geometry, and calculus courses, linear equations are written in either

- **Slope-Intercept Form:** $y = mx + b$
- **Point-Slope Form:** $y = m(x - x_0) + y_0$

Either of these forms are acceptable IF we want to think of y as being a function of x, as these forms tell us how y reacts to changes in x.
If $x_1 = x + \Delta x$ then

$$y_1 = m \cdot (x + \Delta x) + b = (mx + b) + m \cdot \Delta x = y + m \cdot \Delta x$$

In other words, if x increases by a fixed amount then y increases by m times that fixed amount.

Disadvantages of point-slope and slope-intercept forms

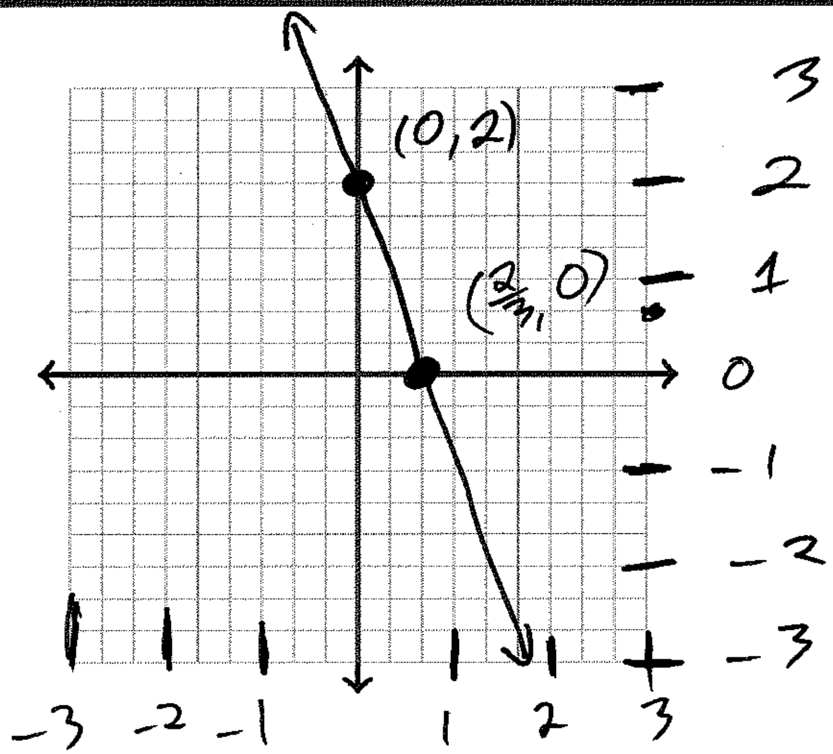
- Point-slope and slope-intercept forms are unable to handle vertical lines

- Point-slope and slope-intercept forms view one of the variables as a function of the other. In other words, the two variables are treated differently, whereas in business applications the variables are generally on equal footing

- Generalizing point-slope and slope-intercept forms to handle more than two variables is awkward.

~~Equation of Line II~~

You know the intercepts, so connect the dots



General Form of a Line

The general form of a line is an expression of the form

$$Ax + By = C$$

To be an equation of a line, we require that at least one of A or B is non-zero.

Example: Given the line $6x + 2y = 4$, find

- **x-intercept** $y=0 \Rightarrow 6x=4 \Rightarrow x=\frac{4}{6}=\frac{2}{3}$
- **y-intercept** $x=0 \Rightarrow 2y=4 \Rightarrow y=\frac{4}{2}=2.$
- **graph**
- **slope**

If $x \nearrow 1$, then $6x \nearrow 6$, so $2y \searrow 6$, so $y \searrow 3$. Slope $= -3$.

In other words, slope $= -\frac{B}{A} = -\frac{6}{2} = -3$

Special Cases of General Form

Vertical Lines Suppose $B = 0$ and $A \neq 0$. The line is then

$$Ax + 0y = C$$

which is equivalent to $x = \frac{C}{A}$, a vertical line.

Horizontal Lines Suppose $A = 0$ and $B \neq 0$. The line is then

$$0x + By = C$$

which is equivalent to $y = \frac{C}{B}$, a horizontal line.

Lines through Origin Suppose $A \neq 0$, $B \neq 0$ and $C = 0$. The line is then

$$Ax + By = 0$$

which is equivalent to $y = -\frac{A}{B}x$, a line through the origin with slope $-A/B$

Generic Case of General Form

Consider a line with general form

$$Ax + By = C$$

in which none of A, B, and C are zero. Then

- **x-intercept** is $(C/A, 0)$
- **y-intercept** is $(0, C/B)$
- **slope of y in terms of x**, $\frac{\Delta y}{\Delta x} = -\frac{A}{B}$
- **slope of x in terms of y**, $\frac{\Delta x}{\Delta y} = -\frac{B}{A}$
- To graph it, plot the x and y intercepts and connect the dots.

Identifying linear functions

A stock analyst is looking at recent stock prices for several companies, and she wonders which of the prices involve a linear trend. which have constant slope?

d1 7 d2 7 d3 7 d4 7 d5

MNO stock:

date	Feb 3	Feb 10	Feb 17	Feb 24	Mar 3
price	\$24.50	\$24.75	\$25.00	\$25.25	\$25.50

.25 .25 .25 .25

Each 7 day ↗ results in \$0.25 ↗. Constant slope, so its linear.

7 7 7 7

PQR stock:

date	Feb 3	Feb 10	Feb 17	Feb 24	Mar 3
price	\$32.00	\$31.00	\$29.00	\$28.00	\$25.00

−1 −2

Different 7 day ↗ result in different \$ changes. Not constant slope Non-linear

7 14 14

STU stock:

date	Feb 3	Feb 10	Feb 24	Mar 10
price	\$80	\$82	\$86	\$90

2 4 4

Linear.

Twice the increase, but twice the time, $\frac{\Delta \text{Price}}{\Delta \text{time}} = \frac{2}{7}$

Interpreting the Coefficients

You manufacture pencils. You observe that you will sell 4000 pencils per month if you charge \$0.50 per pencil, and 3700 per month if you charge \$0.65 per pencil. Let $q(s)$ denote the number of pencil you expect to sell at price s.

$m = \frac{\Delta q}{\Delta s} = \frac{4000-3700}{.65-.5} = -2000.$

- Assuming $q(s)$ is linear, find an expression for $q(s)$.

$q(s) = q_0 + m(s - s_0) \Rightarrow q(s) = 4000 - 2000(s - 0.50)$

$\boxed{q(s) = 5000 - 2000s}$

- In plain english, explain the meaning of the s-intercept, y-intercept, and slope.

Slope $= -2000$ pencils/dollar $\Rightarrow$ Each 1 dollar $\nearrow$ in price $\Rightarrow$ sell 2000 fewer pencils.

See next page $\longrightarrow$

- You decide to make 3800 pencils. What is the highest price you can charge if you want to sell all 3800 pencils?

set $q(s) = 3800$

$3800 = 5000 - 2000s$

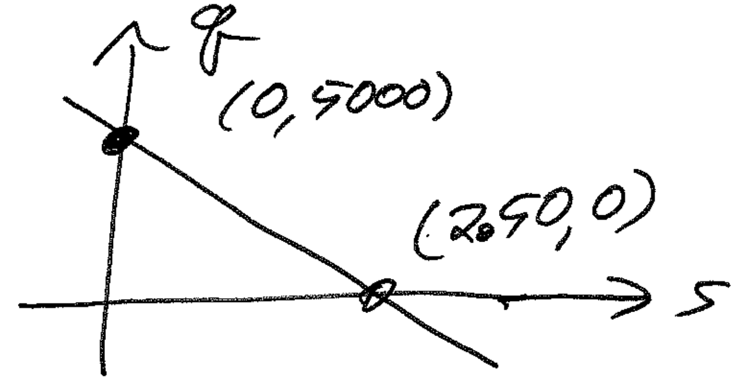
$2000s = 1200$

$s = \frac{1200}{2000} = \$0.60$

y-intercept: 5000.

Even if we give pencils away for free, will only be able to give away 5000.

Regardless of price we charge, we cannot expect to sell more than 5000



s-intercept: set $q = 0$

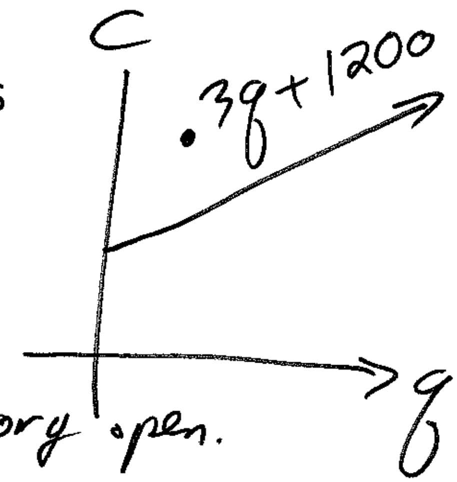
$$5000 - 2000s = 0$$

$$\Rightarrow s = \frac{5000}{2000} = \$2.50$$

Nobody will buy @ \$2.50 or higher.

Cost

Suppose the cost to produce q pencils per month is $C(q) = \$0.30q + \$1200$.



- Find the y-intercept and explain its meaning.

$q = 0 \Rightarrow C = 1200$

Even if we produce no pencils, still costs us \$1200. This is a fixed costs like cost of keeping factory open.

- Find the slope and explain its meaning.

Slope = \$0.30/pencil.

Each additional pencil costs us \$0.30 to make

- Determine the cost to produce 3800 pencils.

$q = 3800 \Rightarrow C(q) = 0.30 \cdot 3800 + 1200 = \$2340$

- Is this a good business plan? NO.

Previous problem ⇒ Get \$0.60 per pencil, 3800 pencils.

So get $(3800)(0.60) = \$2280$.

Costs more (\$2340) than we will earn (\$2280).