Quiz 6

Name: ____________________ Section and/or TA: ____________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Use the Chain Rule to find $\frac{dz}{dt}$ where $z = \sqrt{1+xy}$, $x = t^2 + 1$, and $y = 4t + 12$. No need to simplify.

> **Solution:** Recall $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$ We find: $\frac{\partial z}{\partial x} = \frac{y}{2\sqrt{1+xy}}$, $\frac{\partial z}{\partial y} = \frac{x}{2\sqrt{1+xy}}$, $\frac{dx}{dt} = 2t$, and $\frac{dy}{dt} = 4$. Thus $\frac{dz}{dt} = \frac{ty}{\sqrt{1+xy}} + \frac{2x}{\sqrt{1+xy}} = \frac{ty+2x}{\sqrt{1+xy}}$.

2. (3 points) Find the directional derivative of the function $f(x,y) = x^2y + 4y\sin(x)$ at the point $(0,2)$ in the direction of the **unit** vector $\mathbf{u} = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$.

> **Solution:** $D_u f(x,y) = \nabla f(x,y) \cdot u$,
>
> $$\begin{aligned}\nabla f(x,y) &= \langle f_x(x,y), f_y(x,y) \rangle \\ &= \langle 2xy + 4y\cos(x), x^2 + 4\sin(x) \rangle.\end{aligned}$$
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> At the point $(0,2)$ this gives $\nabla f(0,2) = \langle 8, 0 \rangle$, so
>
> $$\begin{aligned}D_u f(0,2) &= \langle 8,0 \rangle \cdot \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle \\ &= 8 \cdot \frac{1}{\sqrt{5}} + 0 \cdot \frac{2}{\sqrt{5}} \\ &= \frac{8}{\sqrt{5}}.\end{aligned}$$