MA 213 Worksheet #13

Section 14.6
10/9/18

1 Find the directional derivative of f at the given point in the direction indicated by the angle θ.
14.6.5 $f(x,y) = \sqrt{2x+3y}, \quad (3,1), \quad \theta = -\pi/6$

2 (a) Find the gradient of f. (b) Evaluate the gradient at the point P. (c) Find the rate of change of f at P in the direction of the vector $\mathbf{u}$.
14.6.10 $f(x,y,z) = y^2 e^{xyz}, \quad P(0,1,-1), \quad \mathbf{u} = \langle \frac{3}{13}, \frac{4}{13}, \frac{12}{13} \rangle$

3 Find the directional derivative of the function at the given point in the direction of vector $\mathbf{v}$.
14.6.13 $g(s,t) = s\sqrt{t}, \quad (2,4), \quad \mathbf{v} = 2\mathbf{i} - \mathbf{j}$
14.6.15 $f(x,y,z) = xy^2 \tan^{-1} z, \quad (2,1,1), \quad \mathbf{v} = \langle 1,1,1 \rangle$

4 Find the maximum rate of change of f at the given point and the direction in which it occurs.
14.6.22 $f(x,y) = \sin(xy), \quad (1,0)$
14.6.23 $f(x,y,z) = x\ln(yz), \quad (1,2,\frac{1}{2})$

5 *14.6.33* Suppose that over a certain region of space the electrical potential V is given by $V(x,y,z) = 5x^2 - 3xy + xyz$.
(a) Find the rate of change of the potential at $P(3,4,5)$ in the direction of the vector $\mathbf{v} = \mathbf{i}+\mathbf{j}-\mathbf{k}$.
(b) In which direction does V change most rapidly at P?
(c) What is the maximum rate of change at P?

6 Find equations of (a) the tangent plane and (b) the normal line to the given surface at the specific point.
14.6.42 $x = y^2 + z^2 + 1, \quad (3,1,-1)$
14.6.44 $xy + yz + zx = 5, \quad (1,2,1)$

7 *14.6.40* The **second directional derivative** of $f(x,y)$ is

$$D^2_{\mathbf{u}} f(x,y) = D_{\mathbf{u}}[D_{\mathbf{u}} f(x,y)].$$

(a) If $\mathbf{u} = \langle a,b \rangle$ is a unit vector and f has continuous second derivatives, show that

$$D^2_{\mathbf{u}} f = f_{xx}a^2 + 2f_{xy}ab + f_{yy}b^2$$

(b) Find the second directional derivative of $f(x,y) = xe^{2y}$ in the direction of $\mathbf{v} = \langle 4,6 \rangle$.