MA 213 Worksheet #26

Sections 16.5

12/4/18

1 *16.5.1,3* Find (1) the curl and (2) the divergence of the vector field.

(a) $\mathbf{F}(x, y, z) = xy^2z^2\mathbf{i} + x^2yz^2\mathbf{j} + x^2y^2z^2\mathbf{k}$

(b) $\mathbf{F}(x, y, z) = xye^z\mathbf{i} + yze^x\mathbf{k}$

2 *16.5.13,15* Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

(a) $\mathbf{F}(x, y, z) = y^2z^3\mathbf{i} + 2xyz^3\mathbf{j} + 3xy^2z^2\mathbf{k}$

(b) $\mathbf{F}(x, y, z) = z\cos(y)\mathbf{i} + xz\sin(y)\mathbf{j} + x\cos(y)\mathbf{k}$

3 *16.5.23* Prove the identity, assuming that the appropriate partial derivatives exist and are continuous. If f is a scalar field and $\mathbf{F}, \mathbf{G}$ are vector fields, show div($\mathbf{F}$+$\mathbf{G}$)=div$\mathbf{F}$+div$\mathbf{G}$.

4 *16.5.21* Show that any vector field of the form

$$\mathbf{F}(x, y, z) = f(x)\mathbf{i} + g(y)\mathbf{j} + h(z)\mathbf{k}$$

where f, g, h are differentiable functions, is irrotational.

5 *16.5.21* Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = |\, \mathbf{r} \,|$. Verify each of the following identities.

(a) $\nabla \cdot \mathbf{r} = 3$

(b) $\nabla \cdot (r\mathbf{r}) = 4r$

(c) $\nabla r = \mathbf{r}/r$

(d) $\nabla \times \mathbf{r} = \mathbf{0}$

(e) $\nabla(1/r) = -\mathbf{r}/r^3$