MA 213 Worksheet #27

Review for Final!

12/6/18

Chapter 16 Questions: *Taken from Chapter 16 Review, pgs 1148-1149.*

1 (a) Write the definition of the line integral of a scalar function f along a smooth curve C with respect to arc length.

(b) State the Fundamental Theorem for Line Integrals.

(c) What does it mean to say that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path? If you know that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path, what can you say about $\mathbf{F}$?

(d) Suppose $\mathbf{F}$ is a vector field on $\mathbb{R}^3$. Define curl $\mathbf{F}$ and div$\mathbf{F}$.

2 Evaluate the line integral: $\int_C yz\cos(x)ds$, where $C : x = t, y = 3\cos(t), z = 3\sin(t), 0 \leq t \leq \pi$

3 Show by example the following is false: If $\mathbf{F}$ and $\mathbf{G}$ are vector fields and div$\mathbf{F}$=div$\mathbf{G}$, then $\mathbf{F} = \mathbf{G}$.

4 Evaluate the line integral: $\int_C xydx + y^2dy + yzdz$, where C is the line segment from $(1, 0, -1)$ to $(3, 4, 2)$.

5 Show that $\mathbf{F}(x, y) = (1 + xy)e^{xy}\mathbf{i} + (e^y + x^2e^{xy})\mathbf{j}$ is a conservative vector field. Then find the function f such that $\mathbf{F} = \nabla f$

6 Use Green's Theorem to evaluate $\int_C x^2ydx - xy^2dy$, where C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.

Chapter 12-15 Questions:

7 *12* - Find an equation of the plane through $(2, 1, 0)$ and parallel to $x + 4y - 3z = 1$.

8 *12* - Find a vector perpendicular to the plane through the points $A = (1, 0, 0), B = (2, 0, -1), C = (1, 4, 3)$. Now find the area of triangle ABC.

9 *13.3.25* - Find the curvature of $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at the point $(1, 1, 1)$.

10 *13.4.41* - Find the tangential and normal components of the acceleration vector at the given point

$$\mathbf{r}(t) = \ln t\,\mathbf{i} + (t^2 + 3t)\mathbf{j} + 4\sqrt{t}\mathbf{k}, \qquad (0, 4, 4)$$

11 *14.5.21* - Use the Chain Rule to find the indicated partial derivatives.

$$z = x^2 + y^2, \quad x = s + 2t - u, \quad y = stu^2$$

$$\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u}, \qquad \text{when } s = 4, t = 2, u = 1$$

12 *14.7.31* - Find the absolute maximum and minimum values of f on the set D

$$f(x, y) = x^2 + y^2 - 2x$$

D is the closed triangular region with vertices $(2, 0), (0, 2)4$ and $(0, -2)$.

13 *14.8.5* - The following is an extreme value problem with both a maximum and minimum value. Use Lagrange Multipliers to find the extreme values of the function subject to the given constraint.

$$f(x, y) = xy$$
$$4x^2 + y^2 = 8$$

14 *14.4.19* Given that f is a differentiable function with $f(2, 5) = 6$, $f_x(2, 5) = 1$ and $f_y(2, 5) = -1$, use a linear approximation to estimate $f(2.2, 4.9)$.

15 *15.8.27* - Find the volume of the part of the ball $\rho \leq a$ that lies between the cones $\varphi = \pi/6$ and $\varphi = \pi/3$.

16 *15.2.61* Find the average value of $f(x, y) = xy$, over the region D, where D is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 3)$.