MA 213 - Calculus III
Fall 2017
Exam 3 November 15, 2017

## Exam Scores

Do not write in the table below

Name: $\qquad$

Section: $\qquad$

Last 4 digits of student ID \#: $\qquad$

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- All questions are free response questions. Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

| Question | Score | Total |
| :---: | ---: | ---: |
| 1 |  | 10 |
| 2 |  | 10 |
| 3 |  | 10 |
| 4 |  | 10 |
| 5 |  | 10 |
| 6 |  | 10 |
| 7 |  | 10 |
| 8 |  | 10 |
| 9 |  | 10 |
| 10 |  | 10 |
| Total |  | 100 | Unsupported answers may not receive credit.

Free Response. Always show your work!

1. (10 points) Change the order of integration in the following iterated integral:

$$
\int_{0}^{1} \int_{\sqrt{x}}^{1} e^{x^{3} y} d y d x
$$

Do not evaluate the integral.
2. (10 points) Find the average value of $f(x, y)=x y$ over the triangle with vertices $(0,0)$, $(1,0)$, and $(1,3)$.

Free Response. Always show your work!
3. (10 points) Use polar coordinates to find the volume of the solid $E$ inside the cylinder $x^{2}+y^{2}=1$ and the ellipsoid $4 x^{2}+4 y^{2}+z^{2}=64$, and above the $x y$-plane.
4. (10 points) A lamina occupies the part of the disk $x^{2}+y^{2} \leq 1$ in the first quadrant. Find its center of mass, if the density is given by $\rho(x, y)=x+y$.
Note: You may need to use the formulas

$$
\sin ^{2} \alpha=\frac{1-\cos (2 \alpha)}{2}, \quad \cos ^{2} \alpha=\frac{1+\cos (2 \alpha)}{2}
$$

Free Response. Always show your work!
5. (10 points) Find the surface area of the part of the plane $6 x+4 y+2 z=1$ that lies inside the cylinder $x^{2}+y^{2}=25$.
6. (10 points) Evaluate

$$
\int_{0}^{2} \int_{0}^{z^{2}} \int_{0}^{y-z}(2 x-y) d x d y d z
$$

7. (10 points) Compute the volume of the solid bounded by the paraboloid $z=4 x^{2}+4 y^{2}$ and the plane $z=a(a>0)$.

Free Response. Always show your work!
8. (10 points) Find the spherical coordinates $(\rho, \theta, \phi)$ of the point $P$ whose rectangular coordinates are $(-1,1,-\sqrt{2})$.
9. (10 points) In spherical coordinates, the equation $\rho=2 \sin \phi \sin \theta$ represents a sphere. Find the center $C$ and the radius $R$ of this sphere.
10. (10 points) (a) Describe in spherical coordinates the region $E$ that lies between the spheres $x^{2}+y^{2}+z^{2}=4$ and $x^{2}+y^{2}+z^{2}=9$.
(b) Express $x^{2}+y^{2}$ in spherical coordinates $(\rho, \theta, \phi)$.
(c) Write the triple integral

$$
\iiint_{E}\left(x^{2}+y^{2}\right) d V
$$

as an iterated integral using spherical coordinates. Do not evaluate the integral.

