Math 213 Exam 3

Name: ______________________ Section: __________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used other than a one-page "cheat sheet" of notes, formulas, etc., written or typeset on one or both sides of an 8-1/2″ × 11″ sheet of paper. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 6 free-response questions. Please follow these guidelines to receive maximum credit.

- Each question is followed by a space to write your answer. Please write your answer *neatly* in the space provided.
- Show all work to receive full credit on the free response problems. You will be graded on the clarity of your presentation as well as the correctness of your answers.
- Give exact answers, rather than decimal equivalents, unless otherwise instructed (e.g., $\sqrt{2}$, not 1.414).

Question	1	2	3	4	5	6	**Total**
Possible	10	18	18	18	18	18	100
Score							

1. (Coordinate Systems - 10 points) Find:

   (a) (4 points) Find the rectangular coordinates of the point whose spherical coordinates are $(\rho, \theta, \phi) = (4, \pi/6, \pi/4)$

   $x =$ ______

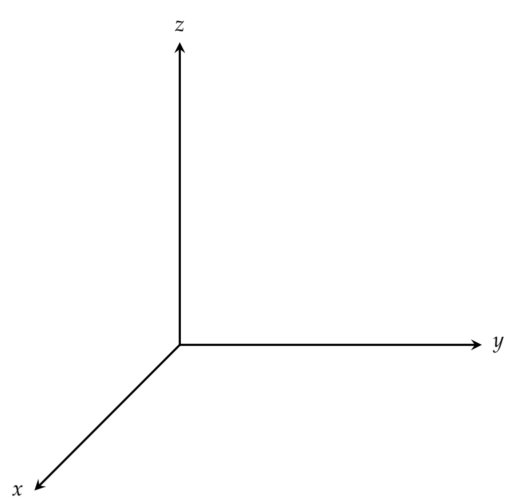
   $y =$ ______

   $z =$ ______

   (b) (6 points) Sketch the solid described in spherical coordinates by the inequalities

   $$0 \leq \rho \leq 2, \quad 0 \leq \theta \leq \pi/2, \quad 0 \leq \phi \leq \pi/2$$

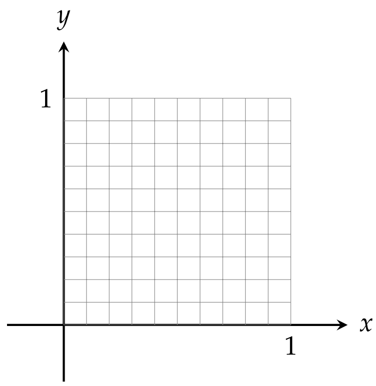
   and describe the solid. Be sure to label intercepts with the x, y, and z axes.



2. (Iterated Integrals -18 points) The purpose of this problem is to compute the iterated integral

$$\int_0^1 \int_x^1 \cos(y^2)\, dy\, dx$$

(a) (6 points) Sketch the region of integration on the axes provided.

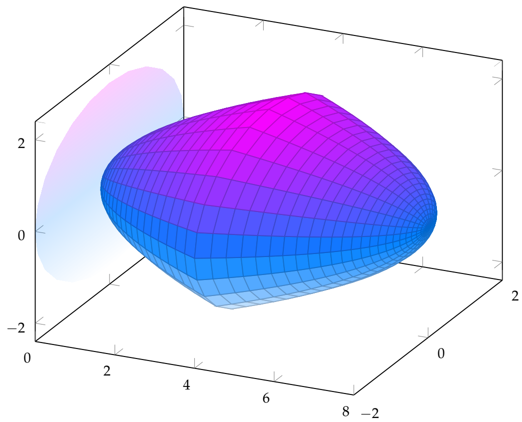


(b) (6 points) Write down an iterated integral equivalent to the given one but with the orders of integration in x and y reversed.

(c) (6 points) Compute the iterated integral using your result from part (b).

3. (Triple Integrals - 18 points) The purpose of this problem is to find the volume of the solid enclosed by the paraboloids $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$.

   (a) (6 points) Describe the region using cylindrical coordinates $x = r\cos\theta$, $z = r\sin\theta$.



$$\_\_\_\_ \le r \le \_\_\_\_$$

$$\_\_\_\_ \le \theta \le \_\_\_\_$$

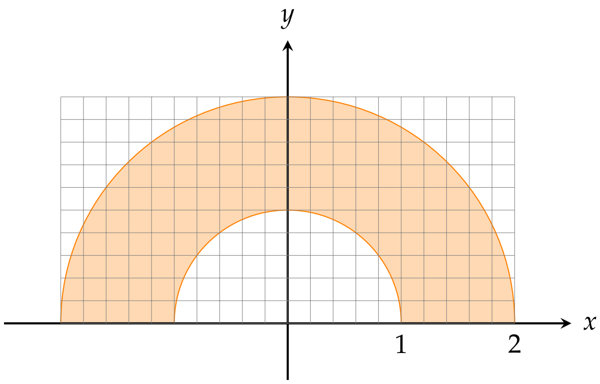
$$\_\_\_\_\_\_\_\_ \le y \le \_\_\_\_\_\_\_\_$$

   (b) (6 points) Write down a triple integral in cylindrical coordinates for the volume of the solid.

   (c) (6 points) Evaluate the triple integral.

4. (Applications of Double Integrals - 18 points) Find the mass of the lamina bounded by the semicircles $y = \sqrt{1-x^2}$ and $y = \sqrt{4-x^2}$ together with the portions of the x axis that join them, assuming that the mass density is $\rho(x,y) = \sqrt{x^2+y^2}$.

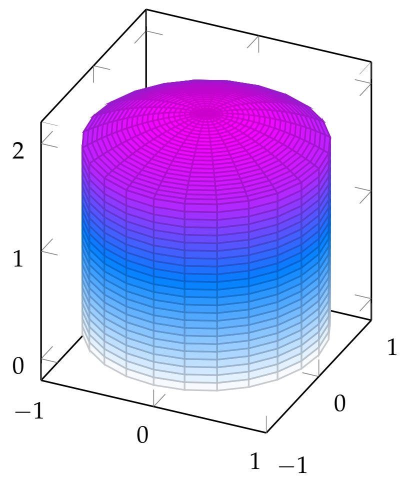
   Hint: Use polar coordinates.



5. (Triple Integrals - 18 points)

The purpose of this problem is to find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

(a) (6 points) Describe the solid in cylindrical coordinates by filling in the table below.



____ $\leq r \leq$ ____

____ $\leq \theta \leq$ ____

____ $\leq z \leq$ ________

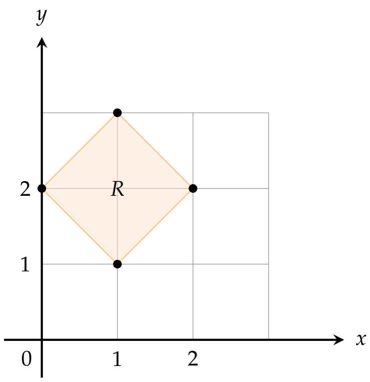
(b) (6 points) Set up the volume integral in cylindrical coordinates.

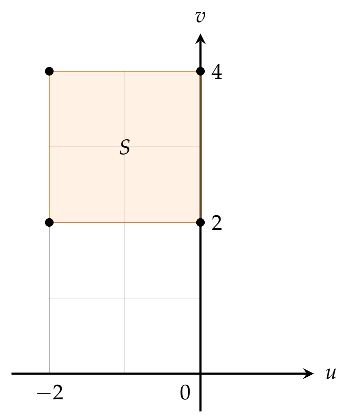
(c) (6 points) Evaluate the integral.

6. (Change of Variables - 18 points) The purpose of this problem is to evaluate the integral

$$\iint_R \frac{x-y}{x+y}\, dA$$

where R is the square with vertices $(0,2)$, $(1,1)$, $(2,2)$ and $(1,3)$ in the xy plane, shown at left.





(a) (4 points) By completing the table below, show that the transformation

$$u = x - y, \quad v = x + y$$

maps R onto the region S shown at right.

x	y	u	v
0	2		
1	1		
2	2		
1	3		

(b) (4 points) Using the equations $u = x - y$, $v = x + y$, solve for x and y in terms of u and v.

(c) (4 points) Find the Jacobian determinant of the transformation from (u, v) to (x, y) found in part (b). Write your answer in the space provided, and be sure to show your work!

$$\frac{\partial(x, y)}{\partial(u, v)} = \underline{\qquad\qquad}$$

(d) (6 points) Use the transformation from part (b), the Jacobian determinant from part (c) and the change of variables theorem to evaluate $\displaystyle\iint_R \frac{x-y}{x+y}\, dA$.