Math 213 Exam 4

Name: ______________________________ Section: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used other than a one-page "cheat sheet" of notes, formulas, etc., written or typeset on one or both sides of an 8-1/2″ × 11″ sheet of paper. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 8 free-response questions. Please follow these guidelines to receive maximum credit.

- Each question is followed by a space to write your answer. Please write your answer *neatly* in the space provided.
- Show <u>all work</u> to receive full credit on the free response problems. You will be graded on the clarity of your presentation as well as the correctness of your answers.
- Give exact answers, rather than decimal equivalents, unless otherwise instructed (e.g., $\sqrt{2}$, not 1.414).

Question	1	2	3	4	5	6	7	8	Free	**Total**
Possible	10	15	10	10	10	15	10	15	5	100
Score									5	

1. (10 points) Consider the points $A(0,0,1)$, $B(-1,0,4)$, and $C(1,2,1)$.

   (a) (5 points) Find a vector perpendicular to the plane that contains these three points.

   (b) (5 points) Find the area of the triangle ΔABC.

2. (15 points) A catapult launches a stone at a height of 10 feet and an angle of $\pi/6$ radians, with an initial speed of 250 feet per second. Recall that the acceleration due to gravity is -32 ft/sec^2.

   (a) (5 points) Write down a vector equation for the position of the stone at time t.

   (b) (10 points) Suppose 300 feet away there is a castle wall 100 feet tall. Does the stone pass over the wall?

3. (10 points) Use the chain rule to find the following derivatives.

$$z = x^2 + x^2 y, \quad x = s + 2t \quad y = st$$

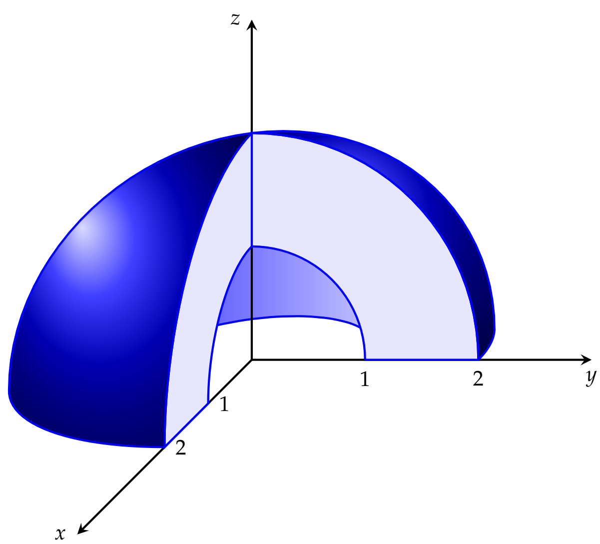
$$\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t} \quad \text{when} \quad s = 1,\ t = 2.$$

4. (10 points) Use Lagrange multipliers to find the extreme value(s) of the function

$$f(x, y, z) = 2x - y + z$$

on the sphere $x^2 + y^2 + z^2 = 6$.

5. (10 points) Set up but do not evaluate the triple integral of an arbitrary continuous function $f(x,y,z)$ in spherical coordinates over the region E shown in the figure below.



6. (15 points) Consider the transformation T from the uv-plane to the xy-plane given by

$$T: \quad x = u + 2v, \quad y = 3u - 3v.$$

(a) (3 points) Compute the inverse transformation T^{-1}.

(b) (4 points) Compute the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ of T.

(c) (4 points) Let S be the rectangle in the uv-plane with vertices $(1,0)$, $(4,0)$, $(4,2)$, $(1,2)$. Show that $T(S)$ is the parallelogram R in the xy-plane with vertices $(1,3)$, $(4,12)$, $(8,6)$, $(5,-3)$.

u	v	$x = u + 2v$	$y = 3u - 3v$
1	0		
4	0		
4	2		
1	2		

(d) (6 points) Use the change of variables formula to compute

$$\iint_R \frac{3x - y}{3x + 2y}\, dA.$$

7. (10 points) (a) (6 points) Find a potential function f for the vector field

$$\mathbf{F}(x,y) = (3 + 2xy^2)\mathbf{i} + 2x^2y\mathbf{j}$$

(b) (4 points) Using the potential function from part (a), find $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the arc of the hyperbola $y = 1/x$ from $(1,1)$ to $(4, \frac{1}{4})$.

8. (15 points) Use Green's Theorem to evaluate $\oint_C ye^x\,dx + 2e^x\,dy$ if C is the rectangle with vertices $(0,0)$, $(3,0)$, $(3,4)$, and $(0,4)$.