MA 213 - Calculus III Exam 3

Spring 2018
April 11, 2018

## Exam Scores

Do not write in the table below

Name: $\qquad$

Section: $\qquad$

Last 4 digits of student ID \#: $\qquad$

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- All questions are free response questions. Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

| Question | Score | Total |
| :---: | ---: | ---: |
| 1 |  | 10 |
| 2 |  | 10 |
| 3 |  | 10 |
| 4 |  | 10 |
| 5 |  | 10 |
| 6 |  | 10 |
| 7 |  | 10 |
| 8 |  | 10 |
| 9 |  | 10 |
| 10 |  | 10 |
| Total |  | 100 | Unsupported answers may not receive credit.

## Free Response. Show your work!

1. (10 points) Evaluate

$$
\iint_{R} \frac{x y^{2}}{x^{2}+1} d A
$$

where $R=\{(x, y) \mid 0 \leq x \leq 1,-3 \leq y \leq 3\}$.
2. (10 points) Change the order of integration in

$$
\int_{1}^{2} \int_{0}^{\ln x} f(x, y) d y d x
$$

i.e. determine $A, B, C, D$ for which

$$
\int_{1}^{2} \int_{0}^{\ln x} f(x, y) d y d x=\int_{A}^{B} \int_{C}^{D} f(x, y) d x d y
$$

3. (10 points) Set up an iterated integral in polar coordinates to compute the area inside the circle $(x-1)^{2}+y^{2}=1$ and outside the circle $x^{2}+y^{2}=1$. Do not evaluate the integral.
4. (10 points) Find the average distance from a point $(x, y)$ of the disk $x^{2}+y^{2} \leq R^{2}$ to its center.

## Free Response. Show your work!

5. (10 points) Find the mass of a lamina occupying the region $D$ in the $x y$-plane bounded by $y=1-x^{2}$ and $y=0$ if the density is $\rho(x, y)=3 y$.
6. (10 points) Find the surface area of the part of the plane $2 x+2 y+z=5$ inside the cylinder $x^{2}+y^{2}=3$.

## Free Response. Show your work!

7. (10 points) Consider

$$
\iiint_{E} z d V
$$

where $E$ is enclosed by

$$
z=0, \quad z=x^{2}+y^{2}, \quad \text { and } \quad x^{2}+y^{2}=4 .
$$

Use cylindrical coordinates to express this triple integral as an iterated integral. Do not evaluate the integral.
8. (10 points) Find the spherical coordinates $(\rho, \theta, \phi)$ of a point whose rectangular coordinates are $(-1, \sqrt{3}, 2 \sqrt{3})$.
9. (10 points) Change to spherical coordinates

$$
\int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \int_{\sqrt{x^{2}+y^{2}}}^{\sqrt{2-x^{2}-y^{2}}} x^{2} y^{3} d z d y d x
$$

Do not evaluate the integral.
10. (10 points) Evaluate

$$
\iiint_{E}\left(1+y e^{x^{2} z}\right) d V
$$

where $E$ is the cube $E=[-1,1] \times[-1,1] \times[-1,1]$. [Hint: Integrate with respect to $y$ first.]

