Quiz 2

Name: ________________________ Section and/or TA: ________________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Consider the vectors $\mathbf{u} = \langle 1, 4, -2\rangle$ and $\mathbf{v} = \langle 2, 3, 1\rangle$.

   (a) (3 points) Find one **unit** vector orthogonal to $\mathbf{u}, \mathbf{v}$.

Solution: The cross product produces two possible orthogonal vectors, depending on the order in which you take the cross product.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} \mathbf{k} = \langle 10, -5, -5\rangle$$

$$\mathbf{v} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 1 \\ 1 & 4 & -2 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 4 & -2 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} \mathbf{k} = \langle -10, 5, 5\rangle$$

These are not unit vectors, so we need to divide by the magnitude:

$$|\langle 10, -5, -5\rangle| = |\langle -10, 5, 5\rangle| = \sqrt{100 + 25 + 25} = \sqrt{150} = 5\sqrt{6}.$$

Then the answer is either $\langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\rangle$ or $\langle \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\rangle$.

   (b) (1 point) Verify mathematically that your solution from part (a) is orthogonal to $\mathbf{u}$.

Solution: Nonzero vectors are orthogonal if and only if their dot product is zero. Then $\langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\rangle$ is orthogonal to $\mathbf{u}$ since

$$\langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\rangle \cdot \langle 1, 4, -2\rangle = \frac{2}{\sqrt{6}} - \frac{4}{\sqrt{6}} + \frac{2}{\sqrt{6}} = 0.$$

Likewise, $\langle \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\rangle$ is orthogonal to $\mathbf{u}$ since

$$\langle \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\rangle \cdot \langle 1, 4, -2\rangle = \frac{-2}{\sqrt{6}} + \frac{4}{\sqrt{6}} - \frac{2}{\sqrt{6}} = 0.$$

(c) (1 point) Find the area of the triangle formed between vectors **u** and **v**.

> **Solution:** The area of the triangle is half the magnitude of their cross product, so $\frac{5\sqrt{6}}{2}$.