Quiz 9

Name: ____________________ Section and/or TA: ____________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. In this problem we will evaluate

$$\iint_R xy\,dx\,dy$$

where R is the parallelogram in the xy- plane with vertices (0,0), (4,1), (2,2), and (6,3).

(a) (2 points) Consider the transformation $T(u,v) = (x,y)$:

$$\begin{aligned} x &= 4u + 2v \\ y &= u + 2v. \end{aligned}$$

Show that the region in the uv- plane corresponding to the parallelogram R under the transformation T is the square $[0,1] \times [0,1]$.

Solution: Note that

$$\begin{aligned} T(0,0) &= (0,0) \\ T(1,0) &= (4,1) \\ T(0,1) &= (2,2) \\ T(1,1) &= (6,3) \end{aligned}$$

So the corresponding region in the uv- plane is $[0,1] \times [0,1]$.

(b) (1 point) Calculate the Jacobian of the transformation T.

Solution: The Jacobian is the matrix

$$\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 2 \end{pmatrix}$$

Its determinant is $(4)(2) - (2)(1) = 6$.

(c) (2 points) Evaluate the integral $\iint_R xy\,dx\,dy$ by making the above change of variables.

Solution: Using the change of variables we can write the integral as

$$\int_0^1 \int_0^1 (4u + 2v)(u + 2v)|6|\,du\,dv$$

$$= 6\int_0^1 \int_0^1 4u^2 + 10uv + 4v^2\, du\, dv$$

$$= 6\int_0^1 \frac{4}{3} + 5v + 4v^2\, dv$$

$$= 6\left(\frac{4}{3} + \frac{5}{2} + \frac{4}{3}\right)$$

$$= 6\left(\frac{31}{6}\right)$$

$$= 31$$