MA 213 Worksheet #22

Section 16.3 & 16.4
04/09/19

1 Determine whether or not $\mathbf{F}$ is a conservative vector field.

16.3.3 $\mathbf{F}(x,y) = (xy + y^2)\mathbf{i} + (x^2 + 2xy)\mathbf{j}$.
14.3.7 $\mathbf{F}(x,y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$

2 Find a function f such that $\mathbf{F} = \nabla f$ and evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.

16.3.12 $\mathbf{F}(x,y) = (3 + 2xy^2)\mathbf{i} + 2x^2y\mathbf{j}$, C is the arc of the hyperbola $y = 1/x$ from $(1,1)$ to $(4, \frac{1}{4})$.
16.3.15 $\mathbf{F}(x,y,z) = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k}$, C is the line segment from $(1,0,-2)$ to $(4,6,3)$.

3 Show the line integral is independent of path and evaluate the integral.
16.3.19 $\int_C 2xe^{-y}dx + (2y - x^2e^{-y})$, where C is any path from $(1,0)$ to (2,1).

4 *16.3.24* Find the work done by the force field $\mathbf{F}(x,y) = (2x + y)\mathbf{i} + x\mathbf{j}$ in moving an object from $P(1,1)$ to $Q(4,3)$.

5 *16.4.1,3* Evaluate the line integral by two methods: (i) directly and (ii) using Green's Theorem.

(a) $\oint_C y^2\,dx + x^2y\,dy$
where C is the rectangle with vertices $(0,0), (5,0), (5,4)$, and $(0,4)$.

(b) $\oint_C xy\,dx + x^2y^3\,dy$
where C is the triangle with vertices $(0,0), (1,0)$, and $(1,2)$.

6 *16.4.7* Use Green's Theorem to evaluate

$$\oint_C (y + e^{\sqrt{x}})\,dx + (2x + \cos y^2)\,dy$$

where C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

7 *16.4.13* Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle y - \cos y, x\sin y\rangle$ and C is the circle $(x-3)^2 + (y+4)^2 = 4$ oriented clockwise.

8 *16.4.17* Use Green's Theorem to find the work done by the force $\mathbf{F}(x,y) = x(x+y)\mathbf{i} + xy^2\mathbf{j}$ in moving a particle from the origin along the x-axis to $(1,0)$, then along the line segment to $(0,1)$ and then back to the origin along the y-axis.