Name: ____________________ Section and/or TA: ____________

1. (2 points) Given that f is a differentiable function with $f(2,5) = 6$, $f_x(2,5) = -1$, and $f_y(2,5) = 1$, use linear approximation to estimate $f(2.1, 4.8)$.

Solution: We have

$$L(x,y) = f(2,5) + f_x(2,5)(x-2) + f_y(2,5)(y-5),$$

i.e.,

$$L(x,y) = 6 - (x-2) + (y-5) = 3 - x + y.$$

Then

$$f(2.1, 4.8) \approx L(2.1, 4.8) = 6 - (0.1) + (-0.2) = 5.7\ .$$

2. (2 points) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, assuming that z is defined implicitly as a function of x and y by the equation $e^z = 2xyz$.

Solution: We differentiate the equation $e^z = 2xyz$ with respect to x assuming that z is a function of x and y:

$$e^z \left(\frac{\partial z}{\partial x}\right) = 2yz + 2xy\left(\frac{\partial z}{\partial x}\right).$$

Solving this for $\dfrac{\partial z}{\partial x}$ gives

$$\frac{\partial z}{\partial x} = \frac{2yz}{e^z - 2xy}.$$

Similarly (or by symmetry),

$$\frac{\partial z}{\partial y} = \frac{2xz}{e^z - 2xy}.$$