Homework 1

STA321 Spring 2009

Due Jan. 29

1. Let $Y$ be a binomial $(n, p)$ random variable with parameter $p$ ($n$ is known). Let $\hat{p} = Y/n$ Show that $(\hat{p})^2$ is NOT an unbiased estimator of $p^2$.

2. Let $X_1, \ldots, X_n$ be iid $\sim$ Bernoulli($p$). Define $\hat{p} = (1/n) \sum X_i$.
   a) Compute the MSE for estimating $p$ with $\hat{p}$. Show $\hat{p}$ is a consistent estimator of $p$. (we did this in class, the MSE is $p(1-p)/n$)
   b) Let $\tilde{p} = (0.5 + \sum X_i)/(n + 1)$. Compute the MSE of $\tilde{p}$ and show $\tilde{p}$ is also consistent.
   c) When $n = 100$ and $p = 0.5$, which estimator ($\hat{p}$ or $\tilde{p}$) is the better estimator in terms of MSE? When $p = 0.1$ and $n = 100$, which is the better estimator?
   d) When $n = 100$, and $p = 0.01$, which estimator is the better estimator in terms of MSE?

3. Let $X_1, \ldots, X_n$ be iid $\sim N(\mu, \sigma^2)$ where both $\mu$ and $\sigma^2$ are unknown, $-\infty < \mu < \infty$, $0 < \sigma^2 < \infty$. (Please note, here the variance $\sigma^2$ is the parameter not the standard deviation $\sigma$)
   a) compute the MSE of estimator $s^2$ as an estimator of $\sigma^2$. ($s^2$ is the so called sample variance).
   b) Let us try a new estimator of $\sigma^2$: $\hat{\sigma}^2 = C s^2$, for some positive constant $C$.

Find the best constant $C$ in the sense that minimizes the MSE.