Notes 2, April, Sta624

Example of MC for continuous state space (cont. random variables). But still discrete time.

1. Transition probability matrix needs to be replaced by a transition probability kernel: $p(x|y)$; which is a density for any given y value.

2. in the following example $p(x|y)$ is taken to be Uniform $(1-y, 1)$ State space is the interval $(0, 1)$.

Example of a MC

$X_0 \sim$ any number between 0 and 1 (or could be from any distribution on (0,1) interval)

$X_1 \sim$ Unif $(1 - X_0, 1)$

$X_2 \sim$ Unif $(1 - X_1, 1)$

......

$X_n \sim$ Unif $(1 - X_{n-1}, 1)$; ...

This is a MC.

If the distribution of X_n is convergent at all (here it does), the stationary distribution must satisfy the following (integral) equation

$$\int p(x|y)f(y)dy = f(x)$$

We may check that distribution $f(x) = 2x$ solves the above equation.

Therefore X_n, as n large, will have a distribution approximately equal to $f(x) = 2x$ for $0 < x < 1$.

Remark: this is the cont. version of the equation $\pi_0 P = \pi_0$ for discrete MC.

Since

$$p(x|y) = \frac{I_{[1-y<x<1]}}{y}$$

We compute, using $f(y) = 2y$,

$$\int_0^1 p(x|y)f(y)dy = \int I_{[1-y<x<1]} 2dy = 2\int_{1-x}^1 dy = 2x\ .$$

Notice here we only used the random variables from uniform distributions and end up with a random variable with density $f(x) = 2x$.

The idea of MCMC is to use an easy $p(x|y)$ in the iteration but end up with a random variable with (complicated) $f(x)$ that we desire.

Suppose
$X_0 \sim$ any number between 0 and 1
$X_1 \sim$ Unif $(0, 1 - X_0)$
$X_2 \sim$ Unif $(0, 1 - X_1)$
......
$X_n \sim$ Unif $(0, 1 - X_{n-1})$,
Find the stationary distribution $f(x)$ of this MC. This will also be the approx distribution of X_n for large n.