

Notes for Sta 624 Stochastic Processes, Spring 2012

The page number etc below is based on 10th ed. of the book. We shall update them as we go along.

Some topics.

Exponential random variables.

Memoryless property, constant hazard.

Minimum of independent exponential r.v.s is still an exponential.

prob($\text{exp1} > \text{exp2}$) etc.

Poisson process.

3 definitions

mean, intensity, and distribution

stationary and independent increments

splitting, combining/merging Poisson processes

arrival times, inter-arrival times

First, try to go through all the examples in the chapter 5.

If you want some more practice problems try problems at the end of chapter 5

like #26, #28, #30, #38, #39, #53, #58, #62

Some Topics about Markov Chain (MC).

P the transition probability matrix.

P^n the n-step transition probability matrix.

States that communicates with each other form a class.

Recurrent/transient of a state, of a class (how to identify?)

A MC with one class is irreducible

For recurrent states:

The stationary distribution π of an irreducible, recurrent MC satisfy

$$\pi = \pi P,$$

and π is a probability.

The continuous state version (time is still discrete) of this is

$$\int p(x|y)f(y)dy = f(x)$$

and $f(x)$ is a density. (no more than this and the example of note2)

Mean time between visits to a recurrent state i (start in i); mean recurrence time = $1/\pi_i = m_{ii}$.

The expected number of transitions (mean time) it take to visit state j , starting from state i is denoted by m_{ij} . [we did not give a formula, but may use the trick of change j to absorbing state, and find the absorption time.]

For transient states:

Mean number of visits to a transient state j , starting from a transient state i (in the same class) before the MC got absorbed is s_{ij} , from the matrix $S = (I - P_t)^{-1}$.

(on page 228) $f_{ii} = f_i$ (on page 191, 192), probability of ever return to state i when starting from i .

(remember for transient states, $f_i < 1$.)

The mean number of visits to state i (starting in i) is $1/(1 - f_i)$. (by the expectation of the geometric distribution).

Therefore we have $1/(1 - f_i) = s_{ii}$. We also have $s_{ii} = 1 + \sum_{n=1}^{\infty} P_{ii}^n$.

For off diagonal values of f_{ij} , we have $s_{ij} = f_{ij}s_{jj}$. From here you can solve for f_{ij} , where f_{ij} is the probability of visiting state j (starting in i).

Important examples;
Random walk, or Gambler's ruin problem. (reflection/absorption boundary, finite/infinite states, symmetric or not, dimension 1 or 2 or 3).
Queuing models. (finite or infinite size)
Genetic example (Hardy-Weinberg on page 203) .
MCMC.
Brownian motions.