University of Kentucky Lexington

Department of Mathematics Technical Report 2000-36

Unconventional Reflexive Numerical Methods for Matrix Differential Riccati Equations [1]

Ren-Cang Li[2]

November 2000

ABSTRACT

Matrix Differential Riccati Equations (MDREs)

$$X' = A_{21} - XA_{11} + A_{22}X - XA_{12}X, \quad X(0) = X_0,$$

where $A_{ij} \equiv A_{ij}(t)$, appear frequently throughout applied mathematics, science, and engineering. MDREs play particularly important roles in optimal control, filtering, estimation, and in two-point linear boundary value problems. In the past a number of unconventional numerical methods that are suited only for time-invariant MDREs have been designed, but despite their special structure, no unconventional methods that are suited for time-varying MDREs have been constructed, except (carefully) re-designed conventional linear multistep methods and Runge-Kutta methods. Implicit conventional methods which are preferred to explicit ones for stiff systems require solving nonlinear systems of equations (of possibly much higher dimensions than the original problem itself for Runge-Kutta methods) which not only pose implementation difficulties but also may be expensive because they require solving non-linear matrix equations which may be costly. We propose new unconventional reflexive methods which are suited for both time-invariant and time-varying MDREs and which requires solving no nonlinear systems but linear ones like one Sylvester equation per time-step or linear matrix systems with the same size as the MDRE. The new methods are semi-implicit and thus have much better stability properties than explicit methods and more importantly, they can be easily tailored for engineering applications, for example, some new methods can be easily coded for large sparse MDREs without much programming complications. The new methods are reflexive and thus allow simple and easily implementable palindromic compositions or extrapolations to achieve highly accurate numerical solutions whenever necessary.

[1]This report, actually finished in *December 1997*, is available on the web at `http://www.ms.uky.edu/~rcli/`.

[2]Department of Mathematics, University of Kentucky, Lexington, KY 40506 (`rcli@ms.uky.edu`). This work was supported in part by the National Science Foundation under Grant No. ACI-9721388 and by the National Science Foundation CAREER award under Grant No. CCR-9875201.