University of Kentucky Lexington
Department of Mathematics Technical Report 05-01

# Sharpness in Rates of Convergence For CG and Symmetric Lanczos Methods[1]

Ren-Cang Li[2]

January 2005

ABSTRACT

Conjugate Gradient (CG) method is often used to solve a positive definite linear system $Ax = b$. Existing bounds suggest that the residual of the $k$th approximate solution by CG goes to zero like $[(\sqrt{\kappa}-1)/(\sqrt{\kappa}+1)]^k$, where $\kappa \equiv \kappa(A) = \|A\|_2\|A^{-1}\|_2$ is $A$'s spectral condition number. It is well-known that for a given positive definite linear system, CG may converge (much) faster, known as superlinear convergence. The question is "*do the existing bounds tell the correct convergence rate in general?*". An affirmative answer is given here by examples whose CG solutions have errors comparable to the error bounds for all $k$.

A similar question for the convergence rate of Lanczos algorithm for symmetric eigenvalue problems is addressed and answered firmly, too. Conceivably examples devised here may be good testing problems for linear system and eigensystem solvers.

[1] This report is available on the web at `http://www.ms.uky.edu/~math/MAreport/`.

[2] Department of Mathematics, University of Kentucky, Lexington, KY 40506 (`rcli@ms.uky.edu`.) This work was supported in part by the National Science Foundation CAREER award under Grant No. CCR-9875201.