

A Note on Hard Cases For Conjugate Gradient Method¹

Ren-Cang Li²

February 2005

revised October 2005

ABSTRACT

The Conjugate Gradient (CG) method is often used to solve a positive definite linear system $Ax = b$. This paper analyzes two hard cases for CG or any Krylov subspace type methods by either analytically finding the residual formulas or tightly bound the residuals from above and below, in contrast to existing results which only bound residuals from above. The analysis is based on a general framework to estimate CG and MINRES residuals for certain linear systems, and the framework may potentially be useful elsewhere.

¹This report is available on the web at <http://www.ms.uky.edu/~math/MAreport/>.

²Department of Mathematics, University of Kentucky, Lexington, KY 40506 (rccli@ms.uky.edu.) This work was supported in part by the National Science Foundation CAREER award under Grant No. CCR-9875201 and by the National Science Foundation under Grant No. DMS-0510664.