

Vandermonde Matrices with Chebyshev Nodes¹

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ABSTRACT

For $n \times n$ Vandermonde matrix $V_n = (\alpha_j^{i-1})_{1 \leq i, j \leq n}$ with translated Chebyshev zero nodes, it is discovered that V_n^T admits an explicit QR decomposition with the R -factor consisting of the coefficients of the translated Chebyshev polynomials of degree less than n . This decomposition then leads to an exact expression for the condition number of its submatrix $V_{k,n} = (\alpha_j^{i-1})_{1 \leq i \leq k, 1 \leq j \leq n}$ (so-called rectangular Vandermonde matrix), bounds on individual singular value, and more. It is explained that how these results can be used to establish asymptotically optimal lower bounds on condition numbers of real rectangular Vandermonde matrices and nearly optimal conditioned real rectangular Vandermonde matrices on a given interval. Extensions are also made for V_n with nodes being zeros of any (translated) orthogonal polynomials other than Chebyshev ones.

It is also discovered that for V_{n+1} with translated Chebyshev extreme nodes, V_{n+1}^T admits an explicit QR-like decomposition as well. This QR-like decomposition also yields similar conclusions to those for V_n with translated Chebyshev zero nodes.

Applications to the study of sharpness in existing error bounds for the conjugate gradient method and the minimal residual method for linear systems and the symmetric Lanczos method for eigenvalue problems are also discussed.

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