Let B be an $m \times n$ $(m \geq n)$ complex (or real) matrix. It is known that there is a unique *polar decomposition* $B = QH$, where $Q^*Q = I$, the $n \times n$ identity matrix, and H is positive definite, provided B has full column rank. If B is perturbed to $\tilde{B}$, how do the polar factors Q and H changes? This question has been investigated quite extensively, but most work so far was on how the perturbation changed the unitary polar factor Q, and very little on the positive polar factor H, except $\|H - \tilde{H}\|_F \leq \sqrt{2}\|B - \tilde{B}\|_F$ in the Frobenius norm, due to F. Kittaneh (*Comm. Math. Phys.*, 104 (1986), pp. 307–310), where $\tilde{Q}$ and $\tilde{H}$ are the corresponding polar factors of $\tilde{B}$. While this inequality of Kittaneh shows that H is always well-behaved under perturbations, it does not tell much about smaller entries of H in the case when H's entries vary a great deal in magnitudes. This paper is intended to fill the gap by addressing the variations of H for a graded matrix $B = GS$, where S is a scaling matrix and usually diagonal (but may not be.). The elements of S can vary wildly, while G is well-conditioned. In cases as such, the magnitudes of H's entries indeed often vary a lot and thus any bound on $\|H - \tilde{H}\|_F$ means little, if any thing, to the accuracy of $\tilde{H}$'s smaller entries. This paper proposes a new way to measure the errors in the H factor via bounding the scaled difference $(\tilde{H} - H)S^{-1}$, as well as how to accurately compute the factor when S *is* diagonal. Numerical examples are presented.

The results are also extended to the matrix square root of a graded positive definite matrix.