A Concept of Material Symmetry in a Structurally Based Theory of Defective Crystals

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In the structurally based theory of defective crystals, which was originally proposed by Davini [1], a crystal state is defined by prescribing three smooth linearly independent lattice vector fields at each point of a continuum. The main focus of this theory is on such objects like Bürger's vector and the dislocation density tensor (ddt.). The reason for this is that since elastic deformations do not change these (and other) objects they provide an intrinsic measure of inelastic changes of a crystal state. One particular type of such 'plastic deformation` is a planar slip mechanism observed when the lattice vector fields remain constant on a plane fibration. Such changes are called neutral deformations, and as showed by Davini and Parry [2], exist only if the ddt. is singular.

In this presentation we focus on crystal states which not only allow for neutral deformations but are such that the corresponding ddt. is uniform (constant) throughout the crystal lattice. It turns out that the crystal state with the constant ddt. has a Lie group structure such that the lattice vector fields are right invariant with respect to the action of this group, Parry [3]. Moreover, as the ddt. is invariant under elastic deformations, one can choose a crystal state (with the given, constant, singular, ddt.) such that the corresponding group composition function is affine. This ensures that the crystal states considered here have a group property over and above the original Lie group structure. This group property is what allows us to say that crystal states constructed in this way provide the material symmetry groups for the corresponding energy function. These material symmetry groups depend, in general, on the ddt.

Assuming that in order to determine the value of the energy at a point of a lattice it is enough to specify point values of the frame and the ddt., we further postulate that the density of the energy function of a state with a uniform dislocation density tensor is also constant. In other words, we postulate that the lattice state with constant ddt. defines symmetry properties of the corresponding energy function. In this talk we discuss to what extend is this assumption compatible with the notion of material symmetry of nonlinear elasticity theory.

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[2] C. Davini – G. P. Parry, *On defect preserving deformations in crystals*, Int. J. Plasticity, **5,** 1989, 337-369.
[3] G. P. Parry, *Uniform rearrangements in defective crystals*, Proc. R. Soc. Lond. A **455**, 1992, 4333-4346.