Nonlocal configurational entities

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Abstract

Any good modern treatment of Classical Mechanics of systems of particles includes among its fundamental notions the concept of *configuration space*, Q, namely, a manifold representing the kinematic nature of the system. Using this notion as a point of departure, the space of velocities, or virtual displacements, is naturally identified with the tangent bundle, TQ. More importantly, the forces (both internal and external) can be rigorously defined as elements of the cotangent bundle T^*Q. That this point of view is not the adopted standard in Continuum Mechanics is due, at least in part, to the fact that the configuration space of a continuous deformable medium is of infinite dimension. Nevertheless, in spite of obvious technical difficulties, its use may lead to important insights, sometimes of surprising simplicity and elegance. This is the case in the study of material bodies that do not abide by the axiom of locality and that, therefore, do not automatically admit the essentially local notion of stress as a tensor. My aim in this paper is to follow the clue of classical finite-dimensional mechanics so as to provide a more general notion of stress. This in itself, although unusual, is not a novel route. The novelty of the present treatment arises from the combination of this first clue with a second one provided by the geometrical local theory of inhomogeneous continua, such as those containing continuous distributions of dislocations. The material response of such bodies can, in many cases of practical importance, be described by assuming the existence of an elastic archetypal material "point", whose response is implanted into the different points of the (not necessarily elastic) body by means of a map that may evolve in time. This viewpoint has been shown to lead naturally to the notion of Eshelby, or configurational, stress, as a material analog of the Piola stress. These concepts are so general that, as will be shown, they are amenable to direct application to any kind of theory, local or not, that can be formulated in similar geometrical terms. Thus, the generalized Piola stress entity arising from the configuration space viewpoint, will find its material counterpart in a generalized Eshelby entity of a similar mathematical nature.