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Title: Multi-scale stress analysis for composite media: estimation of over stressed zones and development of macroscopic failure criteria.

Abstract: New macroscopic objects that characterize the distribution of stress in the presence of highly oscillatory heterogeneous media are developed. To fix ideas the stress tensor inside a multi-phase linearly elastic composite with micro structure of length scale ε is denoted by σ^ε. One considers the square of the equivalent stress described mathematically by the positive definite quadratic form $\Pi(\sigma^\varepsilon)$. The specific nature of the quadratic form is determined by the particular application. For $t > 0$ the Lebesgue measure of the set inside the composite where $\Pi(\sigma^\varepsilon) > t$ is denoted by $\lambda^\varepsilon(t)$. The analysis focus on characterizing $\limsup_{\varepsilon\to 0} \lambda^\varepsilon(t)$. A new macroscopic quantity, called the macrostress modulation function, is introduced. The macrostress modulation function f captures the excursions of the local stress fluctuations about the homogenized or macroscopic stress field. We introduce the distribution function $\lambda(t, f)$ that gives the measure of the set where $f(x) \geq t$. The macrostress modulation can be thought of giving a measure theoretic upper envelope on the oscillatory equivalent stress in the sense that

$$\limsup_{\varepsilon\to 0}\lambda^\varepsilon(t) \leq \lambda(t, f). \tag{1}$$

This inequality is used to assess the size of over stressed zones due to reentrant corners or sharp changes in boundary loading. The theory underlying this estimate is a new homogenization constraint relating the distribution of states for the stress to the macrostress modulation function. This theory is derived within the general context of G-convergence. The homogenization constraint follows from a new local representation formula for the gradient of G-limits with respect to the constituent elastic properties of the composite.