NORMS OF FORCE FUNCTIONALS AND UNIVERSAL STRESS ESTIMATES

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This work considers analytical properties of forces and stresses in continuum mechanics. In particular, for a given force F, we present an exact lower bound for the essential supremum of any stress field that is in equilibrium with F. The lower bound does not take into consideration any specific constitutive relation. Thus, in a procedure for structural optimization, one may looks for an optimal distribution of material properties that will yield this specific minimum.

The basic mathematical tool we use is a generalization of the method used by Federer to present the flat norm of normal currents. In the continuum mechanics interpretation we regard forces on bodies as (vector valued) 0-currents. In Federer's construction, the flat norm of the 0-current is obtained by a minimization process over all 1-currents. It turns out that in the continuum mechanics interpretation the analogs of the 1-current candidates are the stress fields in the body. The expression for the norm of a force one obtains using Federer's procedure is

$$\|F\| = \inf\left\{\|F - \partial S\|^0 + \|S\|^J\right\}$$

where F denotes the force acting on the body and the infimum is taken over all stress fields S. Here, ∂ is the boundary operator, i.e., the adjoint of the derivative operator, so for any vector field w, $\partial S(w) = S(\nabla w)$. It is noted that the stress fields in this expression need not be in equilibrium with the given force F. We consider two particular examples in some detail.

In the first example, $\|\cdot\|^0$ is the dual norm of the C^0-norm of velocity fields and $\|\cdot\|^J$ is the dual of the C^0-norm for tensor fields. In this case $\|F\|$ is the flat norm of the force.

In the second example, $\|\cdot\|^0$ is the L^q norm of the force field relative to a measure that restricts to the volume measure in the interior of the body and to the area measure on the boundary. The $\|\cdot\|^J$-norm is the regular L^q-norm of stress fields and the $\|\cdot\|$-norm is the dual norm of the L^p_1 Sobolev norm. For this case with $p = 1$, the procedure yields the estimate

$$\operatorname*{ess\,sup}_{i,k,x\in B} |\hat{\sigma}_{ik}| = \inf\left\{A \operatorname*{ess\,sup}_{i,x}\{\left|b_i + \sigma_{im,m}\right|, |t_i - \sigma_{im}\nu_m|\} + \operatorname*{ess\,sup}_{i,k,x} |\sigma_{ik}|\right\}.$$

Here, b_i and t_i are the components of the body force and surface force, A is a constant depending on the body only, $\hat{\sigma}_{ik}$ are the components of a stress field that is in equilibrium with the given force and $\|F\| = \operatorname{ess\,sup}_{i,k,x\in B} |\hat{\sigma}_{ik}|$. The infimum is taken over all smooth stress fields σ_{ik} not necessarily in equilibrium with F. The value $\|F\|$ serves as a lower bound for $\operatorname{ess\,sup}_{i,k,x\in B} |\sigma_{ik}|$ for all stress fields that are in equilibrium with F.

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