Thermoelectric effect in superconductivity

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Traditional engineering applications of superconductivity have been limited to the static case because the time varying currents and magnetic fields in superconductors generate thermal energy, degrading their ability to conduct supercurrents. In this talk we investigate a model for non-isothermal, non-equilibrium superconductivity exhibiting energy losses, and examine how these losses lead to the suppression of superconductivity. This model is derived independently by G. Maugin, and K. Miya and S. A. Zhou and is described by a system of partial differential equations based on the time dependent Ginzburg-Landau (TDGL) equation (1), the Maxwell equations (2), and an energy equation (3) such that the Clausius-Duhem inequality holds.

$$\lambda(\psi_t + i\kappa\phi\psi) + (|\psi|^2 + g(T-1))\psi + \left(-\frac{i}{\kappa}\nabla - \mathbf{A}\right)^2 \psi = 0, \tag{1}$$

$$\mathbf{curl}\ ^2\mathbf{A} = \sigma(-\mathbf{A}_t - \nabla\phi - b_1\nabla T) + Re\left\{\psi^*(-\frac{i}{\kappa}\nabla\psi - \psi\mathbf{A})\right\}, \tag{2}$$

$$\begin{aligned} &T(\ln T - c_1 g'(T-1)|\psi|^2)_t - c_2\Delta T \\ &= 2c_1\left[\lambda|\psi_t + i\kappa\phi\psi|^2 + \sigma|-\mathbf{A}_t - \nabla\phi - b_1\nabla T|^2\right] \\ &\quad -2\lambda\kappa b_1 c_1 T Re\left\{i\psi^*(\psi_t + i\kappa\phi\psi)\right\} \end{aligned} \tag{3}$$

The principal unknown fields are the complex valued Ginzburg-Landau order parameter ψ, the magnetic vector potential $\mathbf{A}$, and the temperature T. A significant feature for this model is that it accounts for the interchange of thermal and electro-magnetic energies through Joule heating.

For the main results, we first establish a conservation of energy law implying that the total energy for an insulated superconducting body (thermal and electro-magnetic) is conserved. Through this, we prove global existence and uniqueness of classical solutions provided that thermolectric constant b_1 is sufficiently small. In particular, when we have a classical Ohm's law ($b_1 = 0$), we analyze the large time behavior of the solution and prove that the ω-limit set as $t \to \infty$ consists of equilibrium solutions $\{(\psi_\infty(x), \mathbf{A}_\infty(x), T_\infty)$, where T_∞ is constant$\}$. We also prove that if the electro-magnetic energy of the superconductor is sufficiently large at $t = 0$ then T will rise beyond the critical temperature, T_c, and $\psi \to 0$ as $t \to \infty$. In contrast to the earlier analytic work based on an isothermal model, this work produces markedly different evolutions* in order parameter and temperature by considering non-isothermal electro-magnetic effects in superconductivity.

* A comparison of isothermal and non-isothermal evolutions is illustrated in Fig. 1.

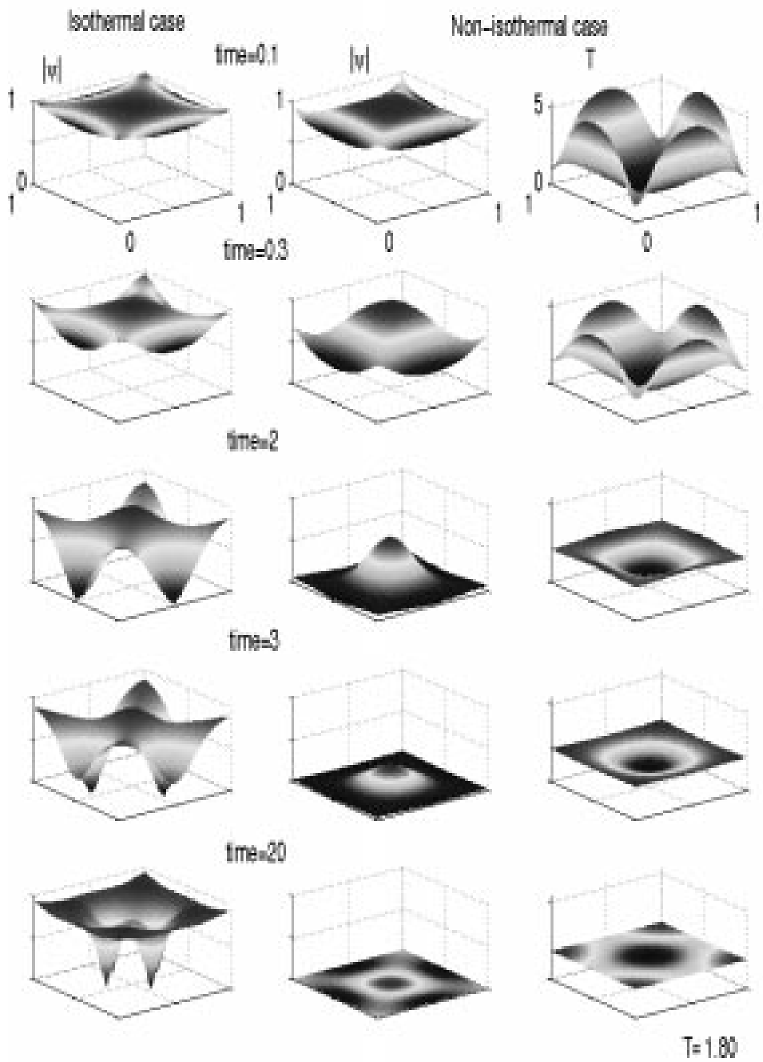

Figure 1: Comparison of isothermal and non-isothermal evolutions.
The first column shows the surface plot of $|\psi|$ for the isothermal case on a unit square. The second ($|\psi|$) and the third columns (T) illustrate the non-isothermal case. In both simulations, the initial conditions for $(\psi_0, \mathbf{A}_0)$ are given by $\psi_0 = 0.8 + i0.6$, $\mathbf{A}_0 = (0, 0)$. The initial condition for T in the non-isothermal case is $T_0 \equiv 0.01$, as is the fixed temperature $\overline{T}$ for the isothermal case. The applied field has magnitude $h = 5$ for $t > 0$. The isothermal simulation evolves to a superconducting steady state with four vortices. For the non-isothermal case, during the evolution hot spots form at the boundary, spreading to the interior. The temperature rises to a constant $T_\infty \approx 1.8$ as t increases, making $g(T_\infty - 1) > 0$, resulting in $|\psi| \to 0$.