# PRACTICE FOR FINAL

**Problem 1:** Let $U \subset \mathbb{R}^n$ be a bounded, connected, $C^1$ domain. Let $c \in C^\infty(\bar{U})$ be a nonnegative, smooth function. Show that the equation

$$\begin{cases} -\Delta u + c(x)u = 0 & \text{in } U \\ u = g & \text{on } \partial U \end{cases} \tag{1}$$

has at most one (classical) solution.

**Problem 2:** Let $U = \{y > 0\} \subset \mathbb{R}^2$. Solve the PDE

$$\begin{cases} u_x + u_y = u^3 & (x,y) \in U \\ u(x,0) = g(x) \end{cases} \tag{2}$$

On what domain does your solution makes sense?

**Problem 3:**

Use the method of separation of variables to find a solution to the backward heat equation

$$\begin{cases} u_t + u_{xx} = 0 & (t,x) \in (0,\infty) \times (0,\pi) \\ u(t,0) = u(t,\pi) = 0 \\ u(0,x) = 4\sin(3x) \end{cases} \tag{3}$$

What is the behavior of the solution as $t \to \infty$?

**Problem 4:**

Use the Fourier transform to solve the Klein-Gordon equation

$$\begin{cases} u_{tt} - \Delta u + u = 0 & t > 0 \\ u(0,x) = g(x), \quad u_t(0,x) = h(x) \end{cases} \tag{4}$$

where $g$ and $h$ are Schwartz functions. Your final answer may include the inverse Fourier transform of a known function.