HOMEWORK 10

1) i) Let $U \subset \mathbb{R}^2$ be the complement of the unit ball, $U = \{|x| \geq 1\}$. Solve the PDE

$$\begin{cases} xu_x + yu_y = 0 & (x, y) \in U \\ u(x, y) = x & (x, y) \in \partial U \end{cases}$$

What is the maximum domain of existence of the solution? Is the solution unique in that domain?

ii) Same question if $U = \{|x| \leq 1\}$ is the unit ball

2) Let $U = \{|x| \leq 1\} \subset \mathbb{R}^2$ be the unit ball. Consider the PDE

$$\begin{cases} u_x + u_y = 0 & (x, y) \in U \\ u|_\Gamma = g & \Gamma \subset \partial U \end{cases}$$

i) What are the characteristic points in ∂U? Does your answer depend on what g is?

ii) Find a set Γ so that the PDE above has a unique global solution for all smooth g.

3) Consider Burger's equation

$$u_t + uu_x = 0, t > 0, \qquad u(0, x) = g(x)$$

i) What are the characteristic points in ∂U?

ii) Solve the equation for $g(x) = -x$. What is the maximum domain of existence of the solution?