HOMEWORK 12 (OPTIONAL)

1) Let $f \in \mathcal{S}$ be a Schwartz function, $h \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$ be constants. Show that

$$\mathcal{F}(e^{ih\cdot x}f(x))(\xi) = (\mathcal{F}f)(\xi - h)$$
$$\mathcal{F}(f(\lambda x))(\xi) = \lambda^{-n}(\mathcal{F}f)(\lambda^{-1}\xi)$$

2) i) Let $f \in C_c^1(\mathbb{R})$. Show that

$$\lim_{\xi\to\pm\infty}(\mathcal{F}f)(\xi) = 0.$$

ii) Show that the same conclusion holds true even if $f \in L^1(\mathbb{R})$. Hint: use i) and approximate f by functions in $C_c^1(\mathbb{R})$.

3) i) Evans 4.7 Problem 8

ii) For u and g as in i), show that the mass is conserved, i.e. for all $t > 0$ we have

$$\|u(t)\|_{L^2} = \|g\|_{L^2}$$