HOMEWORK 2

Please complete any three of the following problems:

1) Evans section 2.5 problem 2

2) i) Show that $\int_{B(0,1)} \ln|x|dx$ is well-defined as an improper integral and compute its value, where $B(0,1)$ is the unit ball in $\mathbb{R}^2$.

ii) Show that there is a constant C so that $\int_{B(0,\epsilon)} |\ln|x||dx \le C\epsilon^2|\log\epsilon|$ for all $0 < \epsilon < \frac{1}{100}$.

3) Show that if $u \in C_c^2(\mathbb{R}^n)$ is harmonic in $\mathbb{R}^n$, then $u \equiv 0$. Hint: Multiply $\Delta u = 0$ by u and integrate

4) Let $f \in C_c^2(\mathbb{R}^3)$ be a fixed function, and let $u = \Phi \star f$ be the solution to Poisson's equation in $\mathbb{R}^3$.

i) Show that there is a constant C_1 so that $|u(x)| \le C_1$ for all $x \in \mathbb{R}^3$. This shows that the solutions to Poisson's equation stay bounded.

ii) Show that actually there is a constant C_2 so that $|u(x)| \le \frac{C_2}{|x|}$ for $|x| > 1$. This shows that the solutions to Poisson's equation actually decay at a rate of at least $|x|^{-1}$ near infinity.