HOMEWORK 4

1) Assume that u is harmonic in $\mathbb{R}^n$, and that it grows at most linearly at infinity, i.e. there are constants R, C_0 and C_1 so that

$$|u(x)| \leq C_0 + C_1|x|, \qquad |x| \geq R$$

Show that there are constants a_i so that

$$u(x) = a_0 + \sum_{i=1}^{n} a_i x_i$$

2) Cacciopoli inequality: If u is harmonic in U, and $\eta \in C_c^1(U)$, then

$$\int_U \eta^2 |Du|^2 dx \leq C \int_U |D\eta|^2 u^2 dx$$

Hint: Multiply your equation by $\eta^2 u$ and integrate by parts.

3) Let U be an open, bounded, connected domain. Show that any two solutions to the Neumann boundary problem

$$\begin{cases} -\Delta u = f & \text{in } U \\ \frac{\partial u}{\partial \nu} = g & \text{on } \partial U \end{cases}$$

must differ by a constant.