HOMEWORK 7

1) Evans section 2.5 problem 15

2) Let $g \in C(\mathbb{R}^n)$ be continuous and bounded, and $\vec{b} \in \mathbb{R}^n$. Find a solution to the equation

$$\begin{cases} u_t - \Delta u + \vec{b} \cdot Du = 0 & t > 0 \\ u(0,x) = g(x) \end{cases}$$

Hint: Try to come up with a function v related to u that satisfies the heat equation.

3) Let $U = (0,1)$. Let u be a solution to

$$\begin{cases} u_t - \Delta u = 0 & t > 0 \\ u(t,x) = 0 & (t,x) \in [0,\infty) \times \partial U \\ u(0,x) = h \in C^\infty(U) \end{cases}$$

Let

$$e(t) = \frac{1}{2} \int_U |u(t,x)|^2 dx, \quad e_1(t) = \int_U |u_x(t,x)|^2 dx$$

i) Show that $e_1(t)$ is a nonincreasing function, and deduce that $\lim_{t\to\infty} e_1(t)$ exists.

ii) Recall from class that $\frac{d}{dt} e(t) = -e_1(t)$. Use i) to show that $\lim_{t\to\infty} e_1(t) = 0$.

iii) Use Cauchy Schwarz and FTC to show that

$$|u(t,x)| \leq (e_1(t))^{1/2}$$

Deduce that $\lim_{t\to\infty} u(t,x) = 0$ for all $0 < x < 1$.