

HOMEWORK 6

1) i) Let $U = [0, 1]^n \subset \mathbb{R}^n$ be the unit cube in \mathbb{R}^n . Show that the inclusion $L^p(U) \subset L^q(U)$ is not compact for any $1 \leq q \leq p$. Hint: Show that the sequence $u_m(x) = \sin(2\pi m x_1)$ is not Cauchy in any space $L^r(U)$.

ii) Show that the inclusion $L^p(U) \subset L^q(U)$ is not compact for any open, bounded set $U \subset \mathbb{R}^n$.

2) Evans section 5.10 problems 15, 18