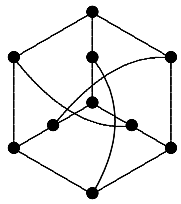
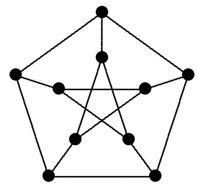
MA 514 Assignment 1. Due Wednesday September 06 at 12:50pm.

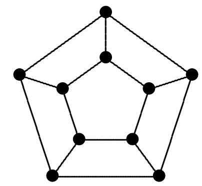
Problem 1A.



G_1



P



G_2

(i) Show that $P \cong G_1$.
(ii) Show that $P \ncong G_2$.
(iii*) (Optional; do this if you're really excited about groups.) Find the automorphism group of the Petersen graph P.

Problem 1B. Suppose G is a simple graph on n vertices that is not connected. Prove that G has at most 36 edges. Can equality occur?

Problem 1C. Show that a connected graph on n vertices is a tree if and only if it has $n-1$ edges.

Problem 1G. Show that a finite simple graph with more than one vertex has at least two vertices with the same degree.

Problem 1I. Let $[n] = \{1, 2, \ldots, n\}$. Let G be the graph with the elements of $[n]^m$ as vertices and an edge between $(a_1, a_2, \ldots, a_m)$ and $(b_1, b_2, \ldots, b_m)$ if and only if $a_i \neq b_i$ for exactly one value of i. Show that G is Hamiltonian.

Problem X1. An *Eulerian trail* in a connected graph G is a path that traverses every edge of G. Show that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.