MA 514 Assignment 4. Due Friday October 27 at 12:50pm.

Problem 4C. Show that if a simple graph on n vertices has e edges, then it has at least $\frac{e}{3n}(4e - n^2)$ triangles.

Problem 4A. Let G be a simple graph with 10 vertices and 26 edges. Show that G has at least 5 triangles. Can equality occur?

Problem 4D. Let $k \geq 2$.

(a) Prove that a k-regular graph of girth 4 has at least $2k$ vertices.
(b) Prove that a k-regular graph of girth $2h$, where $h \geq 2$, has at least $\frac{2(k-1)^h - 2}{k-2}$ vertices.

Problem 4H. Show that a graph on n vertices that does not contain a 4-cycle has at most $\frac{n}{4}\left(1 + \sqrt{4n-3}\right)$ edges.

Problem X4.1.

(a) Show that if G is simple with n vertices and is $K_{2,t}$-free, then it satisfies

$$\sum_{v \in V} \binom{\deg v}{2} \leq (t-1)\binom{n}{2}.$$

(b) Deduce that

$$M(n, K_{2,t}) \leq \frac{1}{2}\sqrt{t-1}\,n^{\frac{3}{2}} + \frac{n}{4}.$$

(c) Show that, given a set of n points in the plane, the number of pairs of points at unit distance is at most $\dfrac{n^{\frac{3}{2}}}{\sqrt{2}} + \dfrac{n}{4}$.