MA 714 - Kostka polynomials and their generalizations Fall 2016

Instructor: Martha Yip
Time: MWF 12:00pm – 12:50pm

Course Description: The Kostka numbers, $K_{\lambda\mu}$, are connection coefficients which appear in the change of basis formula $h_\mu = \sum_\lambda K_{\lambda\mu} s_\lambda$ between the homogeneous basis and the Schur basis for the space of symmetric functions. In the first part of the course, we will examine the role of Kostka numbers in combinatorics and representation theory of the symmetric group. Topics include basic symmetric function theory, tableau combinatorics, and the decomposition of permutation modules into irreducible $\mathfrak{S}_n$-modules.

In the second part of the course, we will study various one- and two-parameter generalizations of the Kostka numbers, sometimes known as Kostka-Foulkes polynomials and Macdonald-Kostka polynomials. Topics include the charge formula of Lascoux and Schützenberger, the Kostant partition function, Hall-Littlewood polynomials, Macdonald polynomials, and Catalan numbers.

Prerequisites: MA 614. Basic knowledge of symmetric function theory is nice to have, but not required. We will review material on symmetric functions as necessary.

Course work: Students will be required to read a section of a textbook or a paper and present a 50 minute talk.

Some References:

(a) W. Fulton, *Young tableaux,* Cambridge University Press, 1997.
(b) A.N. Kirillov, *Ubiquity of Kostka polynomials,* arXiv:math/9912094v4.
(c) I.G. Macdonald, *Symmetric functions and Hall polynomials,* Oxford University Press, 1995.
(d) K. Nelsen and A. Ram, *Kostka-Foulkes polynomials and Macdonald spherical functions,* Surveys in Combinatorics, London Math. Soc. Lecture Note Ser. **307** (2003), 325–370.

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