IMPLICIT EQUATIONS OF MULTIGRADED HYPERSURFACES

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We present a method for computing the implicit equation of a hypersurface given as the image of a rational map $\phi : X \dashrightarrow \mathbb{P}^n$, where X is a product of projective spaces.

In previous works in this direction (cf. [BJ03], [BCJ06], [BD07], [BDD08], [Bot08], et. al.), the approach consists in embedding the space X in a projective space, via a Segre-Veronese map (or a toric embedding when X is a general toric variety). The need of the embedding comes from the necessity of a $\mathbb{Z}$-grading in the coordinate ring of X, in order to study its regularity.

In this talk we give an alternative to this approach: we study the implicitization problem directly, without embedding in a projective space. We use the multihomogeneous structure of the coordinate ring S of X, and we adapt the method of loc. cit. to this setting.

The main motivations for our change of perspective are that it is more natural to deal with the original grading on X, and that the embedding leads to an artificial homogenization process that makes the effective computation slower, as the number of variables to eliminate increases.

Our method can also be generalized to the case where X is a toric variety, and the ring S is the Cox ring of X with the grading given by the Chow group of the variety (cf.[Cox95]).

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