COHEN-MACAULAYNESS VERSUS VANISHING OF $\mathrm{e}_Q^1(A)$

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My talk is based on a joint work [GGHOPV] with Ghezzi, Hong, Ozeki, Phuong, and Vasconcelos. To state the results, let $(A, \mathfrak{m})$ be a Noetherian local ring with $d = \dim A > 0$. Let $\ell_A(M)$ denote, for an A-module M, the length of M. Then, for each $\mathfrak{m}$-primary ideal I in A, we have the integers $\{\mathrm{e}_I^i(A)\}_{0 \le i \le d}$ such that the equality

$$\ell_A(A/I^{n+1}) = \mathrm{e}_I^0(A)\binom{n+d}{d} - \mathrm{e}_I^1(A)\binom{n+d-1}{d-1} + \cdots + (-1)^d \mathrm{e}_I^d(A)$$

holds true for all $n \gg 0$, which we call the Hilbert coefficients of A with respect to I. We say that A is unmixed, if $\dim \widehat{A}/\mathfrak{p} = d$ for every $\mathfrak{p} \in \operatorname{Ass} \widehat{A}$, where $\widehat{A}$ denotes the $\mathfrak{m}$-adic completion of A. With this notation Wolmer V. Vasconcelos posed, exploring the vanishing of $\mathrm{e}_Q^1(A)$ for parameter ideals Q, in his lecture at the conference in Yokohama of March, 2008 the following conjecture.

Conjecture 1 ([GhHV, V])**.** Assume that A is unmixed. Then A is a Cohen-Macaulay local ring, once $\mathrm{e}_Q^1(A) = 0$ for some parameter ideal Q of A.

In my talk I shall settele this conjecture affirmatively. As a direct consequence of the result, one gets that, for a given Noetherian local ring $(A, \mathfrak{m})$ with $d = \dim A > 0$, $\mathrm{e}_Q^1(A) = 0$ for every parameter ideal Q in A, once $\mathrm{e}_Q^1(A) = 0$ for some parameter ideal Q. The next question is, naturally, when the set $\Lambda(A) = \{\mathrm{e}_Q^1(A) \mid Q \text{ is a parameter ideal in} A\}$ is finite, or a singleton. I shall show that the local cohomology modules $\{\mathrm{H}_{\mathfrak{m}}^i(A)\}_{0 \le i \le d-1}$ of A with respect to $\mathfrak{m}$ are all finitely generated, if $\Lambda(A)$ is finite, and eventually that A is a Buchsbaum local ring if and only if $\Lambda(A)$ is a singleton, that is the value $\mathrm{e}_Q^1(A)$ is constant and independent of the choice of parameter ideals Q in A, provided A is unmixed.

References

[GhHV] L. Ghezzi, J.-Y. Hong and W. V. Vasconcelos, *The signature of the Chern coefficients of local rings*, Preprint 2008.

[GGHOPV] Laura Ghezzi, Shiro Goto, Jooyoun Hong, Kazuho Ozeki, Tran Thi Phuong, and Wolmer V. Vasconcelos, *Cohen-Macaulayness versus vanishing of* $\mathrm{e}_Q^1(A)$

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