Lecture 1

$$\int_0^1 x e^x \, dx =$$

$u = x \qquad v = e^x$

$u' = 1 \qquad v' = e^x$

$$= [x e^x]\Big|_0^1 - \int_0^1 e^x \cdot 1 \, dx$$

$$= (1 \cdot e^1 - 0 \cdot e^0) - \int_0^1 e^x \, dx$$

$$= e - e^x\Big|_0^1$$

$e^1 \quad e^0$

$$= e - (e - 1)$$

$$= 1$$

$$\int x \cos x \, dx =$$

$u =$ $\underline{\quad x \quad}$ $\qquad v =$ $\underline{\quad \sin x \quad}$

$u' =$ $\underline{\quad 1 \quad}$ $\qquad v' =$ $\underline{\quad \cos x \quad}$

$$\int \overset{u}{x} \, \overset{v'}{\cos x} \, dx = x \sin x$$

$$- \int \overset{v}{\sin x} \cdot \overset{u'}{1} \, dx$$

$$= x \sin x - (-\cos x) + C$$

$$= x \sin x + \cos x + C$$

$$\int x^4 \ln x \, dx =$$

$u =$ $\underline{\ln x}$ $\qquad v =$ $\underline{x^5/5}$

$u' =$ $\underline{1/x}$ $\qquad v' =$ $\underline{x^4}$

$$\int \overset{v'}{x^4} \, \overset{u}{\ln x} \, dx = \frac{x^5}{5} \ln x - \int \frac{x^5}{5} \cdot \frac{1}{x} \, dx$$

$$= \frac{x^5}{5} \ln x - \int \frac{x^4}{5} \, dx$$

$$= \frac{x^5}{5} \ln x - \frac{x^3}{25} + C$$

$$= \frac{x^5}{5} \ln x - \frac{x^5}{25} + C$$

$$\int x^2 e^x \, dx =$$

(1) $u = \underline{\ x^2\ }$ $\quad v = \underline{\ e^x\ }$

$u' = \underline{\ 2x\ }$ $\quad v' = \underline{\ e^x\ }$

$$\int x^2 e^x \, dx = x^2 e^x - \underbrace{\int e^x \cdot 2x \, dx}_{(2)}$$

(2) $\int$

$\underline{u = 2x}$ $\quad v = \underline{\ e^x\ }$

$\underline{u' = 2}$ $\quad v' = \underline{\ e^x\ }$

(2) $$2xe^x - \int e^x \cdot 2 \, dx = 2xe^x - 2e^x + C$$

u v'

$$\boxed{\int e^x \cos x \, dx} =$$

$u =$ e^x $v =$ $\sin x$

$u' =$ e^x $v' =$ $\cos x$

① $\int e^x \cos x \, dx = e^x \sin x - \underbrace{\int e^x \sin x \, dx}_{②}$

② $\int e^x \sin x \, dx$

$u =$ e^x $v =$ $-\cos x$

$u' =$ e^x $v' =$ $\sin x$

② $\int e^x \sin x \, dx = -e^x \cos x + \boxed{\int e^x \cos x}$

① $\int e^x \cos x \, dx = e^x \sin x - \left(\int e^x \sin x \, dx \right)$ ②

② $\int e^x \sin x \, dx = \left(-e^x \cos x + \int e^x \overset{\cos}{\cancel{\sin}}\, x \right)$

Subs ② into ①

$$\boxed{\int e^x \cos x \, dx} = e^x \sin x + e^x \cos x - \boxed{\int e^x \overset{\cos x}{\cancel{\sin x}} \, dx}$$

$$2 \boxed{\int e^x \cos x \, dx} = e^x \sin x + e^x \cos x + C$$

$$\int e^x \cos x \, dx = \frac{1}{2}\left(e^x \sin x + e^x \cos x\right) + C$$