Math 114 - Integration by Parts

Peter A. Perry

University of Kentucky

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Welcome to Math 114, Spring 2018!

- Bookmark the course web page
  http://www.math.uky.edu/~perry/ma114.s18
- Bookmark the instructor webpage
  http://www.math.uky.edu/~perry/114-s18
- Familiarize yourself with the Canvas Web Page for this course
- Print out and keep in your notebook a copy of the Course Calendar
- Be sure to prepare for recitation tomorrow by reading section 7.1 and beginning to work on Webwork Assignment A1

Unit I: A Toolbox for Integral Calculus

Lecture 1	Integration by Parts
Lecture 2	Trig Integrals
Lecture 3	Trig Substitution
Lecture 4	Integrating Rational Functions, Part I
Lecture 5	Integrating Rational Functions, Part II
Lecture 6	Numerical Integration, Part I
Lecture 7	Numerical Integration, Part II
Lecture 8	Improper Integrals
Lecture 9	(Preview) Sequences
Lecture 10	(Preview) Sequences by Recursion

What Happened in Calculus I

Calculus I introduced two fundamental concepts:

- The *derivative*, a function 'derived' from a given function $f(x)$ that gives its instantaneous rate of change
- The *integral*, a function $F(x) = \int_a^x f(t)\,dt$, a function which gives the net area under the graph of f between a and x

and ended with a remarkable relation between them...

The Fundamental Theorem of Calculus

- (FTC part I) If f is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$
- (FTC part II) If F is any antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$

Practical take-away:

> Any rule of differentiation can be run backwards to give a rule of integration.

Examples

What rule of differentiation do you run backwards to find the following integrals?

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4. $\int \frac{1}{1+x^2} \, dx$

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2. $\int \cos(x)\,dx$
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4. $\int \frac{1}{1+x^2}\,dx$
5. $\int xe^{-x^2}\,dx$

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How does the last example differ from the previous ones?

Integration by Parts

Integration by Parts is a very powerful technique that runs the *product rule* backwards.

First let's remember the product rule:

$$\frac{d}{dx}\left(u(x)v(x)\right) = u'(x)v(x) + u(x)v'(x)$$

and rewrite it as

$$u(x)v'(x) = \frac{d}{dx}\left(u(x)v(x)\right) - u'(x)v(x)$$

According to the FTC, we then get

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

Integration by Parts, Example 1

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

Problem: Find $\int xe^x\,dx$

Fill in the following table:

$u(x) =$ _____ $v(x) =$ _____

$u'(x) =$ _____ $v'(x) =$ _____

Integration by Parts, Example 1

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

Problem: Find $\int xe^x\,dx$

To get rid of the x, pick $u(x) = x$, $v'(x) = e^x$ and fill in the rest:

$u(x) = x$ $\qquad$ $v(x) = e^x$

$u'(x) = 1$ $\qquad$ $v'(x) = e^x$

Integration by Parts, Example 1

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

Problem: Find $\int xe^x\,dx$

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$$\int xe^x\,dx = xe^x - \int 1 \cdot e^x\,dx = xe^x - e^x + C$$

Some people like the following form of the integration by parts rule:

$$\int u\,dv = uv - \int v\,du$$

du is another name for $u'(x)\,dx$
dv is another name for $v'(x)\,dx$.

There is also an integration by parts rule for *definite* integrals:

$$\int_a^b u(x)v'(x)\,dx = [u(x)v(x)]\Big|_a^b - \int_a^b u'(x)v(x)\,dx$$

Switcheroo

For the rest of the lecture, we'll be working a sequence of examples that show how to use integration by parts both for definite and indefinite integrals. I'll be using the document camera to work through these with you. Be sure to take good notes!

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

$$\int_a^b u(x)v'(x)\,dx = [u(x)v(x)]\Big|_a^b - \int_a^b u'(x)v(x)\,dx$$

Parting Shots

- Remember to read the syllabus carefully
- Download and print a copy of the course calendar and keep in your notebook
- Remember to read section 7.1 and download Worksheet 01 for tomorrow's recitation
- Remember to begin Webwork A1
- Be sure you have an *iClicker* account and be ready to use it next week
- Come to my office hours today at 3:15 in POT 755 if you have any questions or concerns about this course