

①

$$\textcircled{1} \frac{5x+10}{x^2-2x-15} = \frac{5x+10}{\underbrace{(x+3)}_{\text{l.f.}} \underbrace{(x-5)}_{\text{l.f.}}} = \frac{A}{(x+3)} + \frac{B}{x-5}$$

$$\textcircled{3} x^3 - 2x^2 = \underbrace{x^2}_{\text{repeated linear factor}} \underbrace{(x-2)}_{\text{linear factor}}$$

Two correct answers

$$\frac{5x+10}{x^2} \quad \frac{5x^2-4x-4}{x^2(x-2)} = \frac{Ax+B}{x^2} + \frac{C}{x-2}$$

(2)

$$\frac{5x^2 - 4x - 4}{x^2(x-2)} = \frac{A_1}{x} + \frac{Ax+B}{x^2} + \frac{C}{x-2}$$
$$= \frac{A_1}{x} + \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

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(4)

$$\frac{2x+1}{x^2+2x+1} = \frac{2x+1}{(x+1)^2}$$

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$$\frac{\boxed{5x+10}}{(x+3)(x-5)} = \frac{\textcircled{1} A}{x+3} + \frac{\textcircled{2} B}{x-5}$$

$$= \frac{\textcircled{1} A(x-5) + \textcircled{2} B(x+3)}{(x+3)(x-5)}$$

$$5x + 10 = A(x-5) + B(x+3)$$

$$x=5: \quad 35 = 0 + 8B$$

$$B = \frac{35}{8}$$

$$x=-3: \quad -5 = -8A$$

$$A = \frac{5}{8}$$

$$\frac{3x+2}{(x^2+x+1)^2} = \frac{A_1x + B_1}{(x^2+x+1)} + \frac{A_2x + B_2}{(x^2+x+1)^2}$$

$$= \frac{(A_1x + B_1)(x^2+x+1) + A_2x + B_2}{(x^2+x+1)^2}$$

$$\underline{0}x^3 + \underline{0}x^2 + 3x + 2 = \left\{ \underline{A_1}x^3 + \underline{(A_1 + B_1)}x^2 + (A_1 + B_1 + A_2)x + (A_1 + B_1 + B_2) \right\}$$

$$0 = A_1$$

$$A_1 = B_1 = 0$$

$$0 = A_1 + B_1 \Rightarrow$$

$$\underline{3}x + \underline{2} = \underline{A_2}x + \underline{B_2}$$

$$A_2 = 3$$

$$B_2 = 2$$

$$x^3 + 2x^2 + x = x(x^2 + 2x + 1)$$

$$= x(x+1)^2$$

$$\frac{5x^2 - 4x - 4}{x^3 + 2x^2 + x}$$

$$= \frac{\textcircled{1} A}{x} + \frac{\textcircled{2} B}{x+1} + \frac{\textcircled{3} C}{(x+1)^2}$$

$$= \frac{A(x+1)^2 + Bx(x+1) + Cx}{x(x+1)^2}$$

$$5x^2 - 4x - 4 = A(x+1)^2 + Bx(x+1) + Cx$$

$$x=0 \quad -4 = A \quad A = -4$$

$$x=-1 \quad 5 + 4 - 4 = -C \quad C = -5$$

$$5 = -C$$

$$\text{Degree } x^2 : \quad 5 = A + B \quad B = 9$$

$$5 = -4 + B$$

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$$\int \frac{5x^2 - 4x - 4}{x^3 + 2x^2 + x} dx =$$

$$\int \left( -\frac{4}{x} + \frac{9}{x+1} - \frac{5}{(x+1)^2} \right) dx$$



7

$$\frac{2x+4}{x(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+1}$$

$$= \frac{A(x^2+x+1) + (Bx+C)x}{x^2+x+1}$$

↓

$$\underline{0}x^2 + \underline{2}x + \underline{4} = \underline{(A+B)}x^2 + \underline{(A+C)}x + \underline{A}$$

$$A=4$$

$$A+C=2$$

$$A+B=0$$

$$A=4$$

$$C=-2$$

$$B=-4$$

8

$$\int \frac{2x+4}{x^3+x^2+x} dx =$$

u-substitution

$$\int \left( \frac{4}{x} + \frac{-4x-2}{x^2+x+1} \right) dx$$